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mathematical

physical

chemical

2nd edition

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Jan ȳwierczek

and the memory of the precursors of Braille mathematical notations in the country

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and Andrzej Galbarski***

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A bit of history

The search for a way of communication between blind and sighted people has a very long history, reaching back to antiquity. In his collection *Disputationum Tusculanarum*, Cicero gives the names of several enlightened blind people - Democritus, Appius Claudius, Aufidius, Diodotus, who aroused general admiration and respect of their contemporaries. In transmitting their knowledge, they certainly used the help of lectors and scribes. We can only assume that they attempted to become independent of third parties by searching for a means of communication without the participation of sight.

The first documented attempts to develop a script accessible to the blind date back to the 14th century. At that time, a blind professor at a university in Iraq, Zain-Din Al Amidi, developed a unique type of convex script with which he marked the volumes of his extensive book collection and took short notes.

European universities also employ blind scholars, whose activities certainly encouraged the search for a means of recording and transmitting information. An additional stimulus was the invention of printing at the end of the 15th century, thanks to which the book became widely available to sighted people.

Francesco Lucas of Saragossa (1517) proposed various versions of the idea of engraving the letters of the alphabet on wooden tablets. In his opinion, the blind could learn the shape of the letters by touching them with their fingers. Erasmus of Rotterdam (1528) has a different idea. He also suggests engraving the letters, but on a hard material, such as an ivory or metal plate. According to him, the blind should run the stylus along the outline of the concave letter until he memorizes the muscular movement of the hand. When he masters this activity, he will be able to write on paper by hand.

It is worth noting here a characteristic feature of every fully-fledged writing system. The information should also be readable by the person writing, something that many ideas for writing for the blind lacked. The technique of writing also plays a significant role. Engraving the shape of letters or embossing them is possible, but quite difficult, slow and complicated.

The Jesuit Lan Terzi came up with an interesting initiative, proposing in 1670 a writing system that was a combination of lines intersecting at right angles and dots carved on paper arranged in individual fields. In this recording system, a blind person could not only write, but also read the information themselves. Lan's introduction of a dot and frame to handwriting was an innovative solution that later played an important role.



Lana Terzi Writing System

Nicholas Saunderson, a mathematician and physicist who was blind in childhood and a professor at Cambridge University, after learning Lan's system, developed boards on which he presented information using pinheads. As Diderot reports, Saunderson was able to use his boards to solve complicated mathematical problems, he could control himself and eliminate any mistakes he noticed.

Many blind people had mastered techniques that enabled them to record information. Most of them, however, used cipher symbols. Only a few had mastered and used the Latin alphabet. All of them belonged to a privileged class. The education of the blind was not organized in any way.

Significant changes in this field occurred in the 18th century. The new trends and theories concerning education that were emerging at that time became the reason for searching for methods of educating the blind. The pioneer was the Frenchman Valentin Haüy, a man with a strong social attitude. After successful attempts to teach the beggar François Leseur, in 1784 he founded the first school in history for the blind. Valentin Haüy's experiment found many imitators. More schools were established to undertake education

blind. Centers were established in England, in many European capitals, and also across the Atlantic. Also in Poland, at the Institute for the Deaf and Dumb, which had existed since 1817, a branch for blind.

All schools struggled with the same problem. They searched hard for a way of writing accessible to the blind. However, the attempts made were based on using the way of writing used by the sighted, which created insurmountable difficulties for the blind.

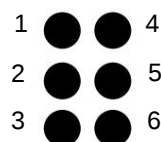
During this period, artillery captain Charles Barbier developed a military cipher script that could be read by touch even in the dark. Like Lan Terzi in the previous century, Barbier introduced a raised dot to encode the written character. Different ciphers were created for different recipients. The basis of the Barbier writing character is a vertical rectangle made up of two rows of six dots. The letters and sounds of the French language are encoded phonetically in this system. Barbier, having developed a suitable ruler for punching characters on paper, presented his invention in 1821 at the Institute for the Blind in Paris, offering it for use by schools.

The project of the new script, introduced to the Institute as a test, was met with an enthusiastic reception from the pupils, and a more skeptical reception from the management and teaching staff. The pupils could easily write themselves, do schoolwork, take notes and read them quickly. However, the phonetic method of writing made it impossible to use orthography, and there was also a lack of punctuation and numbers. A significant disadvantage of Barbier's writing, despite the large number of raised dots used, was the small number of combinations of signs and symbols.

An interesting opinion on Barbier's proposal was expressed by a special commission of the French Academy of Sciences (represented by Lacépède and Ampère), which, after examining the writing system, stated in a special report: "Ordinary writing is an art that appeals to the sight, while that invented by Charles Barbier is an art that appeals to the touch."

Some of the more talented and ambitious students of the institute sought their own way to improve the writing. This group included the young Louis Braille, who was admitted to the Institute as a ten-year-old boy in 1819. Having familiarized himself with Barbier's writing, Braille intuitively felt that this dotted system, which deviated from conventional letters, held a great future for proper writing for the blind. In 1824, he loyally presented his plan to reform the system to Captain Barbier, but he reacted negatively to the proposal. At that time, fifteen-year-old Louis began to introduce radical changes.

In the first stage he reduced the Barbier sign to six dots, arranging them vertically in two rows of three, from which he created a six-point system giving the possibility of 64 combinations of the number and mutual arrangement of dots.



He arranged the characters thus obtained into six series of ten, and allocated the first four series to the Latin alphabet and letters with French diacritics.

The fifth and sixth series were devoted to punctuation, numbers, etc.

The logical development of this system is that the first ten letters of the alphabet from **a** to **j** are represented by combinations of dots 1, 2, 4, 5. The next series of letters are created by successively adding first the third dot, then the third and sixth, and finally the sixth. The shape of a Braille letter is felt by a blind person immediately with the fingertip, without counting the individual dots.

After a few years, thanks to his ingenuity and ability for abstract thinking, Louis Braille, who was then a tutor and music teacher at the Institute, expanded his system to adapt it to mathematical notation, musical notation, various languages, special symbols, etc.

The actual date of the creation of Braille writing can be considered to be 1837, when the second edition of the Braille manual describing the principles of dot writing was published. The system presented in it, with a simple structure, easy, clear, and above all open, adaptable also to foreign languages, based solely on six dots, was already practically crystallized.

Dot Braille in Poland

Braille was known in the Institute for the Deaf in Warsaw. Due to the reluctance and lack of understanding of the problem in 1864-1865, after a few years, Braille teaching was abandoned. Klein's script was used.

1878 At the Institute for the Blind in Lviv, Braille dot writing was introduced into teaching, adapted to the Polish language by Alexander Schönfeld, a blind Polish emigrant in Paris.

1890 Braille was adapted to the Polish language and reintroduced to the Institute in Warsaw.

1925 The first national congress of teachers of special schools. Based on the so-called "Bydgoszcz system" in use, the Polish Braille alphabet was established.

May 24, 1934 Decree of the Ministry of Religious Denominations and Public Enlightenment approving the Braille dot system for the Polish language developed by Elybieta Róya Czacka and Teresa Landy according to the Monnier numbering and the system of Polish Braille abbreviations. November 4, 1981.

Order of the Minister of Education and Upbringing on the introduction of the modified Polish alphabet in the Braille system and orthographic Braille abbreviations in special schools for the blind and visually impaired.

1994 Andrzej Galbarski, ***A manual for copying books in Braille***, First edition, Polish Association of the Blind, Recording and Publishing House, Warsaw 1994.

2003 Guide ***Rules for the use of partially modified Braille notation used in humanistic publications. A guide for teachers, Braille*** instructors, publishers of Braille publications and people ***using Braille*** ZG PZN, Warsaw 2003.

Dotted Braille in mathematical notation 1929–1937

The work of the international commission on the development of a unified system of Braille notation for mathematics.

1941 Colonel Stafford ***International Braille Code for Mathematics and Chemistry*** England. The proposed coding system required further work.

1955 Helmut Epheser ***International Braille Mathematical Notation for the Blind*** Germany, Hannover-Marburg 1955.

A compact and coherent writing system was developed. It was introduced in many countries.

1967 Dr Helmut Epheser ***International Braille Mathematical Notation*** translated and edited by Maria and Andrzej Adamczyk, Main Board of the Polish Association of the Blind, Warsaw 1967.

1986, 1992 Helmut Epheser ***International Braille Mathematical Notation for the Blind*** Germany, Hannover-Marburg, 1986 – Braille version, 1992 – black-print version. Modified notation of 1955. New symbols were introduced and the system for notating multi-level expressions was changed.

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BRaille SIGNS

Below are all the characters of the dotted script in the series arrangement proposed by Ludwik Braille. The scheme includes the letters of the Latin alphabet, punctuation marks and other characters of universal use. In addition, the characters assigned to the Polish national letters are given.

and b c d and f g h j
@abcdefghi series 1

k l m n p q r s t
jklmnopqrs series 2

at in x and with with
tuwxy % \$ Z] \ series 3

and y y y y s In
0 1 2 3 4 5 6 7 8 v series 4

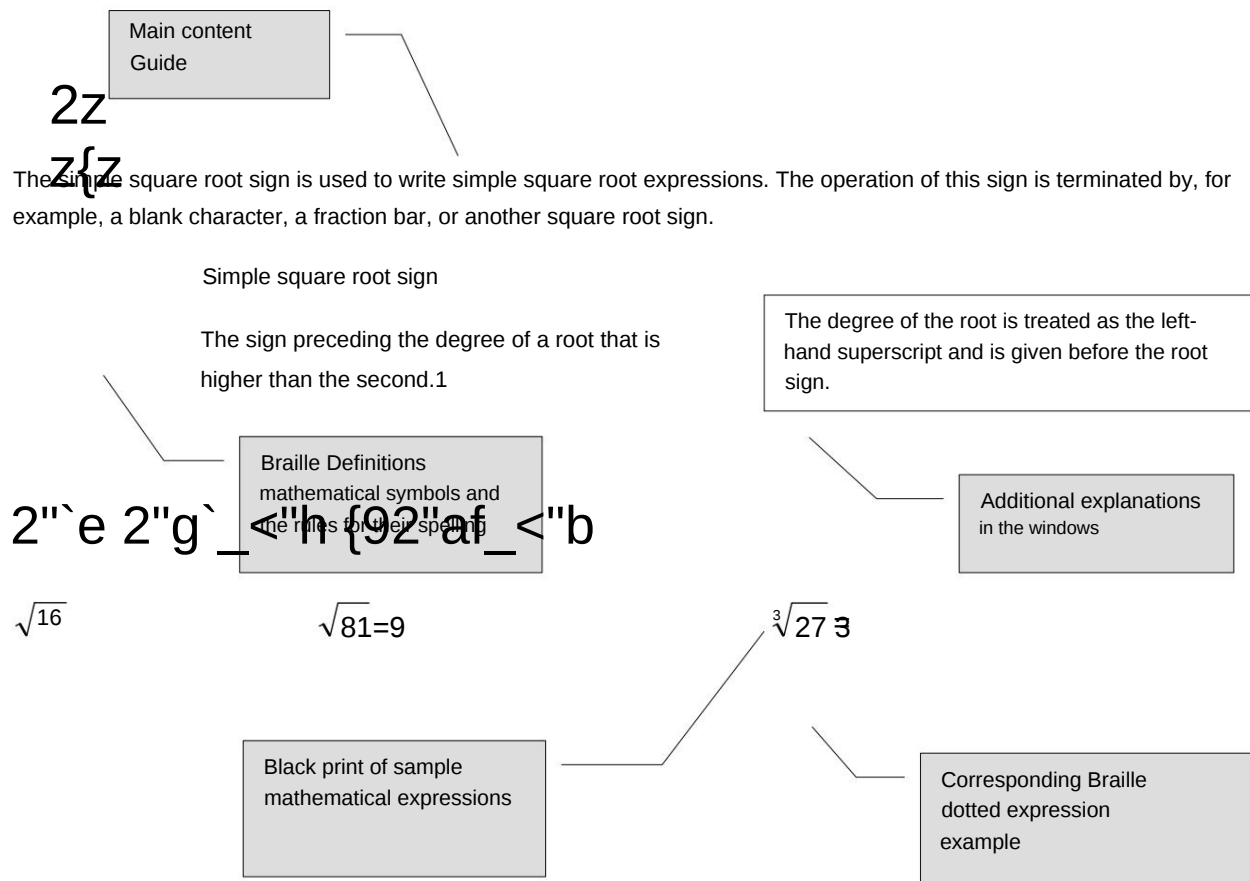
, + : 9 . > * < ') (? ! () " y " ;
sign a poem sign numbers sign capital letter sign italics
series 5

!={?/"#^ ; series 6

apostrophe coupler empty sign
dot
- , & _

ENTRY

The methods of presenting the varied information in the Guide are shown in the fragment below.



The definitions of Braille mathematical symbols and the rules of spelling require a more detailed explanation. The number of Braille symbols limited to 64 forces precise and consistent application of specific rules in mathematical notation. The same symbol can have many meanings. An important role in Braille notation it plays a sequence of signs. Particularly important information is also expressed by the absence of a sign, (space, break). In such a case, in the notation of expressions, one very often speaks of the so-called "empty character". For this purpose, the Guide uses two special characters in the Braille character definitions:

22 an indicative character replacing any Braille character,
blank character (space, blank, no character).

An indicative sign ^{With} makes it easier for sighted people for learning the rules of mathematical Braille notation. This sign is not the so-called "six-dot" marked in the Guide as \$.

The above characters used in the definitions clearly define the "neighborhood" of the defined symbols.

Sequence of signs

Depending on the position of the "blank sign" in mathematical Braille notation, 6 groups of signs are distinguished.

	Sign Definition	Space before character	Space after character
Group A	2	YES or NO	Takes into account the definition of character groups A ŷ, Bŷ, Cŷ .
Group B	* —	YES	
Group C	from+	NO	
Group Aŷ	7	Takes into account the definition of character groups A, B, C.	YES or NO
Group Bŷ	+ —		YES
Group Cŷ	0z		NO

For the sake of clarity, only one character from each group is given in the above list.

A complete definition includes specifying the sequence of characters before and after the symbol being defined.

Sign Definition	Space before character	Space after character
z0z	NO	NO
z5_	NO	YES
_n<z	YES	NO
7	YES or NO	YES or NO
6l_	YES or NO	YES

In the notation of mathematical expressions there may be cases of sequence of signs in which the consistent use of the "empty sign" according to the rules given above leads to a contradiction.
This issue is discussed in detail in the chapter of the Guide – **Mutual position of signs.**

3 The symbols in the list are real mathematical symbols. Their detailed definitions, meanings and rules applications are discussed in later chapters of the Guide.

NUMBERS

In writing Arabic numerals in dotted Braille, the characters of the first series of the alphabet are used, preceded by **a numeric sign (number sign)**.

"z - numeric sign, numeric sign

Cardinal numerals

1	2	3	4	5	6	7	8	9	0
"`	"a	"b	"c	"d	"e	"f	"g	"h	"i

Ordinal numbers

Ordinal numbers are written by placing a dot after the number.

first	second	third	fourth	fifth
"` - "a-	"b-	"c-	"d-	
sixth	seventh	eighth	ninth	feeding
"e-	"f-	"g-	"h-	"i-
tenth		twenty eighth		
"`i-	"ag-			

Integers

6	68	-682
"e	"eg ,eg	

Multi-position number notation

2,584,537,028	"a-dgc-dbf-iag4
2584537028	"adgcdbfiag

The record is very clear
using the dot character `.

The same number written without
the dot is less readable.

You cannot use a blank character (space) as a separator for groups of digits in multi-digit numbers.

4 In Braille dot writing, only a comma is used as a symbol separating the integer part from the decimal part (chapter of the Guide – **Decimal numbers**). Therefore, in Braille writing, there will be no ambiguity, which sometimes occurs in black print writing, where a decimal point is sometimes used.

Decimal numbers

In decimal numbers, the integer part and the fractional part are separated by a

7.29 comma +. "f+ah
 0.072 "i+ifa
 50,347,296 "di+bcf-ahe
 0.333... = 0.(3) "i+bbb---_<"i+1b?

The use of the dot symbol in multi-digit numbers, as in the case of integers, improves readability.

Percentages and per mille

Percentages and permille in Braille dots are marked with the following symbols:

z"i(' – percent with"i((– per mille
 0.25 = 25% 48% = 480%
 "i+ad_<"ad"i("cg"i(_<"cgi"i((

Roman numeral notation

Roman numeral	Arabic script
And #h	1
V #u	5
X #in	10
L #k #b	50
C	100
D #c	500
M #l	1000

Examples of Roman numerals:

III (3)5 IV (4) XLII (42) CDLXXXIV (484)

#hhh #hu #wkhh #bckwwwu

MDCCCXXXVII (1837)

#lcbbbbwwwuhh

5 The Arabic equivalents of the numbers are given in brackets.

ALPHABET SIGNS

In dot Braille, the same characters are used to write the letters of different alphabets.

In addition, the same character can represent a lowercase or uppercase letter. Special key characters called **alphabetic characters allow for unambiguous notation**.

Latin letter notation

The letters of the Latin alphabet are used to write algebraic expressions, units, symbolically designate physical quantities, in the notation of chemical reactions, and in many other cases.

The following key symbols are used to denote Latin letters:

' '&z lowercase latin letter sign

' '#z capital Latin letter sign

and b In

&` &a &v

AND B IN

#@ #a #v

Alphabetical characters precede the character for the appropriate letter.

Greek Letter Writing

The letters of the Greek alphabet are primarily used to represent angles. They can also be found in various branches of mathematics, physics, and chemistry.

The following key symbols are used to denote Greek letters:

;z lowercase greek letter character

^z capital Greek letter sign

ÿ' ÿ ÿ

;` ;a ;v

AND B Oh

^@ ^a ^V

Emphasis Printing

bold print

italics

simple printing

A print type character can only be used in combination with an alphabet character. It is followed by an alphabet character and a letter.

Characters

M

^#L

Capital Latin letter
in italics

P

##ABOUT

Bold large
latin letter

ÿ

#^r

Bold large
Greek letter

F

=#e

Handwritten capital
Latin letter

Greek alphabet

alpha ;` ^`	beta ;a ^a	gamma ;f ^f
delta ;c ^c	Epsilon ;d ^d	zeta ;y ^y
eta ;4 ^4	theta ;3 ^3	jota ;h ^h
kappa ;j ^j	lambda ;k ^k	mu μ
nu n	xi ;w ^w	omicron ;n ^n
o ;o ^o	rho ;q ^q	Sigma ;r ^r
tau ;s ^s	upsilon ;t ^t	phi ;e ^e
chi ;% ^%	psi ;x ^x	omega ;v ^v

Rules for using alphabet characters and bold print6

Alphabetical characters must be used in the following situations to eliminate ambiguity in the notation:

1. Before a letter replacing a mathematical expression and appearing separately in the text mathematical, e.g.

The numbers a , b , c and d form an arithmetic progression.

#khbyax_&`+_&a+_&b_h_&c
svnqy'0_bh0f_`qxsldsxbyx-

The letters a , b , c , d denoting specific numbers are preceded by a lower-case Latin letter.
The conjunction "and" is not preceded by any key character.

6 The rules given are not of a philological nature, but refer to Braille mathematical notation.

2. Before the first letter of a mathematical expression 3. 7.

Before the letter of the first series of the Braille alphabet that is a variable in a mathematical expression, so that this letter was not read as another digit.

2ac

"a&`b

Omission of lowercase Latin letter character in expression 2ac will cause it to be read as the number 213.

213

"a`b

4. Once an alphabet or print character is used in a mathematical expression, it remains valid until the end of that expression. expression or to cancel it with a character of another alphabet or type of printing.

41abC

5KLM

6ÿÿÿ

äÿÿÿ

"c`&`a#B "d#JKL "e;`af ;`a^v

Since most texts are written in simple (regular) print, the simple print character (as opposed to bold or italic) is most often omitted. If it is necessary to show a type of print other than simple, then the print character should precede the alphabetic character.

The combination of two characters (print and alphabet) means that the first one is a print character and the second one is an alphabet character.

abcd

#&`abc

ABCD

^#`abc

5. Alphabetical characters may be used redundantly in algebraic expressions and formulas if there is a risk that the type of letter may be misinterpreted, e.g. due to insufficient experience of the recipient of the notation (e.g. a student).

ÿ + ÿ + ÿ = 180°

;`\_\*;a\_\*;f\_<"`gi(

In = Ult

U = IR

#v_<#t#h&s #t_<#h#q

RECORDING UNITS – BASICS⁹

Braille dot units are preceded by a key unit sign, also known **as a titer sign**.

6z '– name sign, unit sign

1m

1km

m
5 —
sec

230V

"`6l "`6jl "d6l7r10 "abi6#U

Chapter 7 of the Guide – **Algebraic expressions**.

8 Chapters of the Guide – **Latin letter notation, Greek letter notation, and emphasis print characters**.

9 The issues of recording units are discussed in the chapter of the Guide – **Units and operations on units**.

10 The fraction bar symbol 7 is used in the notation of this unit. This symbol is discussed in the **Fractions chapter of the Guide**.

2min

5N

5.2m²

"a6lhm "d6#M

"a+d6l/:11

Units written in full are not preceded by a name sign.

One meter

1 meter

5 newtons

#ldcdm_ldsq "`_ldsq "d_mhtsnm'/v

The name sign is not used for currency names and abbreviations. The currency name or abbreviation should be separated from the number by a space.

2 gold

2 PLN

2 PLN

"a_y'1nsd "a_y'1 "a_#O#K#m

5 PLN 50 gr

5.50 PLN

2 €

"d_y'1_"di_fq "d+di_y1 "a_!d

DATE AND TIME RECORDING

Save the date

Dates should be written in the order of their components in black printing, separating the component parts of the record with dots, slashes or hyphens - as in flat printing - preceding the first of them with a numeric sign. When writing, upper digits (first series) are used. It is permissible to write in which the numeric sign precedes each component of the date.

15/03/2002

"d-ib-aiia "hg.ig.ae or "d-"ib-"aiia

98/08/26

"aiia,ib,`d or "hg."ig."ae

2002-03-15

or "aiia,"ib,"`d

There are also records using Roman numerals.¹²

15/02/2011

"d_#hh_"ai`

Time Notation

When notating time, the full hour is written with a cardinal numeral (first series characters). Minutes are written directly after a dot (third dot) or after a colon with characters of the same series (minutes are not preceded by a numeral sign).

020

05:40

18.25

"i-ai "id9ci "`g-ad

11 The notation uses the exponent sign 12 $\frac{1}{2}$. This sign is discussed in the **Powers and Pointers chapter of the Guide**.
Chapter of the Guide – **Roman numeral notation**.

SIGNS OF ACTIONS AND RELATIONS

Signs of action

$+ _ * z -$	addition	addition and subtraction
$_, z \pm$	subtraction	
$_ *, z m$	plus-minus	
$_, * z \bullet _ - z$	minus-plus	
$\bullet z z \times$	dot in space notation	multiplication
$_ ' z : _ . z$	dot in the record without a space	
$_ 7 _ z 7 z$	cross	division
	division sign	
	fraction bar in full fraction notation	
	fraction bar in shorthand fraction notation	

Note:

- The signs of operations in Braille dot notation belong to the group of signs that must be preceded by a space – an empty character.
- The exception is **the multiplication sign**, which can be written in two ways – with or without a space.
- In Polish mathematical notation, the following has been left $\cdot$, as a division sign (colon). In notation Marburg from 1986 the division sign is the sign $\div$.
- The fraction line notation is discussed in detail in the Fractions chapter of the Guide.

$5 + x$ "d_*&w

$67 : 14$ "ef_."`c

$24.6 + 2 - 4.8$ "ac+e_*"a_,"c+g

in the

notation with a dot and a space

in the

notation with a dot without a space

$12 \bullet 3$

"`a\_-"b "`a-"b

- The absence of a space before the operation sign (except **the multiplication sign**) is a serious notation error.

The operation $5 + x$ written in Braille dots incorrectly without a space "d*&w means $\frac{5}{6}x$, because the first three characters are "d* it's a fraction $\frac{5}{6}$.

Signs of

=	the	equals
≠	relation	does not equal
≡	$_<z _<z$	is equal to identity
≇	$_<<z _<<z$	does not equal identity
:=	$_9<z$	equals by definition
≈	$_>>z$	is approximately equal to
>	$_n+z$	greater
≥	$_n<z$	greater than or equal to
⋈	$_)n+z _nN+z$	no bigger
≫	$_8-z _8<z$	much bigger
<	$_)8-z _88-$	smaller
≤	z	less than or equal to
⋈		no smaller
≪		much smaller

$$7 - 4 = 3$$

$$2.54 \approx 3$$

"f_, "c_<"b "a-"d_8-"cf_."b

BRACKETS

Mathematical

(...)	brackets $1z---z?$	round
[...]	$Zz---z \setminus 8z---$	square
{ ... }	$zn \ Zz---z \& \setminus$	brace
∆ ... ∆		Triangular

$$(14 - 5) + 7 - [1 + 2(5 + 3)]$$

1" `c\_, "d?\_ \* "f\_n+, Z" ` _ * "a1"d_ *
* "b? \

$$2 + \{4 - [5 + (6 - 2)] + 3(6 + 4)\} = 2 + \{4 - [5 + 4] + 30\}$$

"a_ * 8" c_ ', Z" d_ * 1" e_ ', "a? \ _ *
* "b1" e_ * "c? n_ < "a_ * 8" c_ ', Z" d_ *
* "c \ _ * "b in

Brackets for non-mathematical text

Brackets for non-mathematical text are used for additional explanation. They are sometimes called literary brackets. In black print, they appear as round brackets.

(...) $\langle Z \dots Z \rangle$ literary brackets for non-mathematical text

$5 - 3 = 2$ (because $2 + 3 = 5$)

"d_,"b_<"a_<an_"a_*"b_<"d&<

[1]

Unless it leads to misunderstanding, the ampersand before the opening or closing bracket is omitted.

(...) $\langle Z \dots Z \rangle$

However, this character should not be omitted when the end of the bracket immediately follows a number.

Omitting the & sign before the closing bracket in expression [1] will completely change its content.

"d_,"b_<"a_<an_"a_*"b_<"d<

This expression will be read as

$5 - 3 = 2$ (because $2 + 3 = \frac{5}{7}$)¹³

ABSOLUTE VALUE - MODULE

| ... | $!KZ \dots Z^{\wedge}$ absolute value, module

$|-5| = 5$

!K,"d^_<"d

$-|- (7 + 4)| < |(4 - 7)|$

,!k,1"f_*"c?^_8-!k1"c_, "f?^

DIVIDERS OF NUMBERS

$\ddot{y} !K \ddot{y})!K$ is a divisor

$\diagup$ is not a divisor

$5 | 25$ (5 is a factor of 25)

"d_!K"ad or "d!k"ad

$5 \nmid 27$ (5 is not a factor of 27)

"d_)!K"af

¹³ The notation of fractions is discussed in the chapter of the Guide – *Fractions*.

SEPARATE NUMBERS FROM PUNCTUATION MARKS

The following punctuation marks

: 9 _ : _ ? > _ !	colon
;	semicolon
< _ + _	question mark
	exclamation mark
	closed quotation mark
)	closed bracket for non-mathematical text ¹⁴
,	comma

belong to a group of characters used in mathematical notation, which must be written without any space on the left side. A sixth point is placed before a punctuation mark after a number. This rule does not apply to the notation of a date.¹⁵

Correct spelling using the sign		When a sign is accidentally omitted	
12;	"`a&:	"`a:	$\frac{12}{2}$
12?	"`a&>	"`a>	$\frac{12}{5}$
12!	"`a&*	"`a*	$\frac{12}{6}$

Correct notation of numbers and punctuation marks is very important when giving the coordinates of points, writing number ranges, sets, etc.

$P = (3.5)$	#o _ < 1 "b&+ _ "d?
$P = (3,2;5,4)$	#o _ < 1 "b+a&: _ "d+c?
$X = \{1, 2, 3, 4\}$	w _ < 8 "`&+ _ "a&+ _ "b&+ _ "cn
$(3.2; 5.4 \ddot{y})$	1 "b+a&: _ "d+c&\
$A = \ddot{y} 2, 5)$	#` _ < Z - "a&+ _ "d?

The use of punctuation marks can cause ambiguity not only in the notation of numbers. The use

The & sign in the example below before the comma immediately following the closing curly brace eliminates the ambiguity – a curly brace followed by a comma, or a greater-than sign?

$B = \{2, 5, 6\},$	#a _ < 8 "a&+ _ "d&+ _ "en&+
--------------------	------------------------------

¹⁴ The case of this sign has already been discussed above in the chapter of the Guide – *Brackets for non-mathematical text*.
Chapter 15 of the guide – *Recording the date*.

TRANSFERRING PARTS OF MATHEMATICAL EXPRESSIONS TO THE NEXT LINE

When moving parts of mathematical expressions to the next lines, the following rules apply:

1. Whenever possible, the part of a mathematical expression that does not fit in one line should be transferred using a mathematical operation sign or an equals sign, which is placed at the end and repeated at the beginning of the next line.

$$2 + \{4 - [5 + (6 - 2)] + 3(6 + 4)\} = 2 + \{4 - [5 + 4] + 30\}$$

"a_*8"c_',Z"d_*1"e_,"a?_*_*"b1"e_*"c?
n_<"a_*8"c_',Z"d_*_*"c_*"bin

2. A mathematical expression can also be moved where a blank character occurs. In such a case case, dot 6 is used as a carry-over sign, placing it in the place of the empty square where the pattern was interrupted. The & sign indicates that the pattern does not end on a given line.

$$2 + \{4 - [5 + (6 - 2)] + 3(6 + 4)\} = 2 + \{4 - [5 + 4] + 30\}$$

"a_*8"c_',Z"d_*1"e_,"a?_*_*"b1"e&_*"c?
n_<"a_*8"c_',Z"d_*_*"c_*"bin

3. If a mathematical expression is interrupted at a place where there is no empty square, then as the ! character is used to denote a carry over . This character must be repeated at the beginning of the next line.

$$2 + \{4 - [5 + (6 - 2)] + 3(6 + 4)\} = 2 + \{4 - [5 + 4] + 30\}$$

"a_*8"c_',Z"d_*1"e_,"a?_*_*"b1! !"e_*"c?
n_<"a_*8"c_',Z"d_*_*"c\&_*"BIN

ALGEBRAIC EXPRESSIONS

To denote variables occurring in algebraic expressions, appropriate alphabetic characters are used. Braille notation also precisely defines the rules for their use.¹⁶

In addition, the following rules apply when writing algebraic expressions:

1. Once an alphabetic character is placed in an expression, it remains in effect until the end of the expression or until it is canceled by another alphabetic character.
2. An alphabetic character is required before the first letter of a mathematical expression.
The alphabetic sign should also be placed before a letter denoting a mathematical expression and appearing separately in the mathematical text. This issue is discussed in the chapter of the Guide – **Rules for the use of alphabetic signs and distinguished printing (rule 1.)**.

$$3a + b + c \quad [2]$$

"b&`_*a_*b

The lowercase Latin letter & before the variable ` in the Braille notation of expression [2] was used only once – in accordance with the first rule.

$$3x + y + \quad [3]$$

z "b&w_*x_*y

The expression [3] which is very similar in black print to expression [2] creates different problems in Braille. The variables **a, b, c** belong to the first series of the Braille alphabet and, if the alphabetic sign is omitted, can be read as numbers in some cases. In the case of variables x, y, z this ambiguity does not occur.

Omitting **the lowercase Latin** & sign before the variable w in expression [3] will not cause ambiguity in the reading. According to the second rule, however, this sign should be included.

The application of the principles discussed above is illustrated by the following examples.

$$3a + 2b + 4c \quad [4]$$

"b&`_*"a&a_*"c&b

In the Braille notation of the expression [4] it was necessary to use the lower-case Latin letter & before each variable.

$$3x + 2y + 4z \quad [5]$$

"b&w_*"ax_*"cy

In the Braille notation of the expression [5], the lower-case Latin letter & is used only once – before the variable w.

$$2aC + 4Bc - 3.2BD \quad "a\&\#b_*"c\#a\&b_,"b+a\#ac$$

$$2abc \ddot{y}3BCD \ddot{y}BD \quad "a\&`ab_,"b\#abc_,ac$$

$$2abc - 3BCd - bd \quad "a\&`ab_,"b\#ab\&c_,ac$$

$$2abc \ddot{y}3\ddot{y}\ddot{y} \quad "a\&`ab_,"b;\`af$$

3. Alphabetical characters may be used redundantly (e.g. capital letter character, Greek lowercase letter character) in algebraic expressions and formulas if there is a risk that the type of letter may be misread, e.g. due to insufficient experience of the addressee (e.g. student). Therefore, the following is also correct:

$$2aC + 4Bc - 3.2BD \quad "a\&\`#b_*"c#a\&b_,"b+a#a#c$$

$$2abc \ddot{y}3BCD \ddot{y}BD \quad "a\&\`ab_,"b#a\#b\#c_,\#a\#c$$

$$2abc - 3BCd - bd \quad "a\&\`ab_,"b#a\#b\&c_,"ac$$

$$2abc \ddot{y}3\ddot{y}\ddot{y} \quad "a\&\`ab\_,"b;\`;a;f$$

Notation of sets and numerical sets

$A \#$	set A
$N \## M \## b$	set of natural numbers
C	set of integers
$R \## q$	set of real numbers
$In \## V$	set of rational numbers
$IW \## hv$	set of irrational numbers
$c + \## b * 17$	set of positive integers
$R^- \## Q,$	set of negative real numbers
$\emptyset$	empty set

Signs of relations and operations on

$\text{sets } \ddot{y} \text{!d } \text{!d}$	you should
$\ddot{y} \quad \ddot{y} _1-z$	you shouldn't
$_1-z \ddot{y} ?+z _)?+z$	is contained in, is included in, is a subset of
$\ddot{y}$	is not contained, is not contained, is not a subset
	contains
$\nsubseteq$	does not contain

17 A so-called "mark" was used to mark a set. This issue was discussed in the chapter of the Guide - **So-called "marks"**.

$\dot{y}$	$_2-z \ddot{y}$	sum of sets
$_/-z _0-z$		product of sets
$\backslash \times _ 'z$		difference of sets
$\%2z \ddot{y}$		Cartesian product of sets
$\ddot{y} \ \%/z$		indexed sum of sets
R		indexed product of sets
$N \dot{y} \ C \dot{y} \dot{y} \dot{y} \dot{y}$	$###m_1-##B_1-##v_1-##q$	
$IN \dot{y} \ IW$	$##V_)1-##HV$	
$R \dot{y} \ N$	$##Q?+##M$	
$2 \ddot{y} \ C \ ^+$	$"a_!d##b^*$	or $"a!d##b^*$
$C = A \dot{y} C$	$B = D \setminus C$	
$\#b_<\#_2-\#b \#a_<\#c_0-\#b$		

According to the rules of using alphabetic characters¹⁸, the above expressions can also be written as follows:

$C = A \dot{y} C$	$B = D \setminus C$
$\#b_<_2-b$	$\#a_<c_0-b$

U $\dot{y}$ N Ai

$\%20\&h!d##m4\#`0\&h \ 19$

$\dot{y} \ i\dot{y}N \ Ai$

$\%/0\&h!d##m4\#`0\&h$ **Other symbols
used in set algebra**

$\dot{y} \ z) \dot{y} \ z)) \min$	prime 20
"M	encore 21
	minimum

Chapter 18 of the Guide – **Rules for using alphabet characters and distinctive printing**, rule 4.

19 Guide Chapter – **Projection Technique**

20 The symbol is discussed in the Guide Chapter – **So-called “marks”**.

21 As above.

max	"w	maximum
sup	6rto	supremum
inf	6hme	infimum
$\ddot{y}$	$_2+z \ddot{y}$	or 22
$_/+z i \ddot{y} _ <+z$		
$_ <+z$		if and only if
$\ddot{y}$		this (it follows from here)

$$B = \{x : x \in \mathbb{N} \text{ and } x < 7\}$$

#a_<8&w9_w_!d##M_h_&w_8-"fn

$$B = \{x : x \in \mathbb{N} \text{ and } x < 7\}$$

#a_<8&w9_w_!d##M_/_+&w_8-"fn

$$w = \frac{\ddot{y} \times \ddot{y}}{\ddot{y}} p \ddot{y} C \frac{p}{q} \ddot{y} q \ddot{y} C \{1, 0\}$$

##v_<8&w9_w_<o7p_/_+o_!d##b_/_+!23
!&p_!d##b_0-8"inn

$$(\ddot{y}; a) = \{x : x \in \mathbb{R} \text{ and } x < a\}$$

1,"\$: _&`?\_<8w9\_w\_!d##q\_h\_&w8-`n24

$$x \in A \setminus B \text{ if and only if } (x \in A \text{ and } x \notin B)$$

&w_!d#`_0-#a_vsdcx_h_sxkjin_vsdcx
fcx_1&w_!d#`_h_&w_)!d#a?

$$x \in A \setminus B \text{ if } (x \in A \text{ and } x \notin B)$$

&w_!d#`\_0-#a\_<+1&w\_!d#`_/_+! !&w_)!d#a?

$$A = \{x : x \in \mathbb{C} \text{ and } 0 < x < 5\}$$

#`_<8&w9_w_!d##b_h_"i_8-w_8-"dn

22 The symbol is discussed in the *Mathematical Logic chapter of the Guide*.

23 The fraction bar symbol used in the dependency $\frac{7}{}$ is discussed in the Fractions chapter of the Guide.

24 The infinity symbol used in the relationship ∞ is discussed in the *Borders chapter of the Guide*.

$$a, (b) \in A \times B \Rightarrow (a \in A \wedge b \in B)$$

$$1 \& _a? _!d\# _ \#a_ < + 1 \& _!d\# \& / + \&a_!d\#a?$$

$$A \subseteq X$$

$$\# _1 - \# \text{in}$$

$$A^c = X \setminus A$$

$$\# _) _ < \# w _ 0 - \# _$$

complement of set A to set X

$$\text{If } X = \{1, 2, 3, 4, 5, 6\},$$

$$\# \text{id} \% 'dkh _ \# w _ < 8 " \& + _ "a \& + _ "b \& + \& \\ "c \& + _ "d \& + _ "en \& + 25$$

$$A \subseteq X \text{ and } B \subseteq X,$$

$$\# _1 - \# W _ h _ \# a _ 1 - \# W +$$

$$A = \{1, 2, 3, 4, 5\} \text{ and } B = \{4, 5\}, \text{ then:}$$

$$\# _ < 8 " \& + _ "a \& + _ "b \& + _ "c \& + _ "dN \\ h _ \# A _ < 8 "c \& + _ "dn \& + _ sn9$$

$$A \subseteq B = \{1, 2, 3, 4, 5\},$$

$$\# _2 - \# a _ < 8 " \& + _ "a \& + _ "b \& + _ "c \& + \& "dn \& +$$

$$(A \cup B)^c = \{6\},$$

$$1 \# _2 - \# a?) _ < 8 "en \& +$$

$$(A \setminus B)^c = \{4, 5, 6\},$$

$$1 \# _0 - \# a?) _ < 8 "c \& + _ "d \& + _ "en \& +$$

$$B \setminus A = \emptyset,$$

$$\# a _ 0 - \# _ < \% n \& +$$

$$A \subseteq B = \{4, 5\}.$$

$$\# _ / - \# a _ < 8 "c \& + _ "dn -$$

25 The example uses the & sign. Explanation in the chapters of the Guide – *Transferring part of a mathematical expression to next line*, and *Separating numbers and punctuation marks*.

Graphical representation of intervals

In mathematics lessons, there is often a need to present an interval in graphical form. In print flat, intervals are marked as rectangles extending over the appropriate fragment of the number axis, and the types of extremes are marked with circles. An empty circle means that the given extreme does not belong to the interval, while a full circle indicates an extreme that belongs to the interval.

Example:

$$A = [2; 5)$$



Using Braille characters, such a range can be represented graphically as follows:

"a "d
_ \$<<<<w

The first line of the notation corresponds to the number axis, while the second line contains a graphical representation interval, where the \$ sign in denotes the end belonging to the interval (equivalent to a full circle), symbolizes the non-belonging extremity (equivalent to an empty circle in a flat drawing). Between . The extremity numbers on the number line, it is optimal to place three signs as symbols should be placed centrally as possible under the corresponding number entries on the number line.

If the interval is infinite, we do not write any symbol on the infinity side of the number line, nor do we mark the extreme point in any way.

Examples:

$$(-\infty; 5)$$

"d
<<<<w

$$(-\infty; 10 + \infty)$$

"`and _ \$<<<<

If it is necessary to place two or more compartments in one drawing, we place them graphical representations on separate lines, below the number axis:

$$\begin{array}{c} "a "b "d " " _ \$ < < < < < < < w \\ \langle \rangle 5,2 \\ _ w < < < < < < < < \$ \quad (11.3) \end{array}$$

The letter symbols for intervals are written on the same line as the graphical representation of the interval (before or after it), leaving at least a one-character space.

Example:

$$\begin{array}{c} ,,b "i " "f \\ \# _ w < < < < < < < < w \quad (-3.17) \\ < < < < < < < < \$ _ \# a \quad (\ddot{y} \ddot{y} 0, \rangle \end{array}$$

The graphical representation of intervals allows the student to easily locate the area where intervals overlap (the so-called intersection, product), determine the sum and differences of these sets. An important advantage of this notation is the possibility of its quick execution by the student himself, e.g. when solving tasks on a test.

PROJECTION TECHNIQUE

Braille dot matrix has specific features that distinguish it from many other ways of recording information.

- Braille is read with fingers, and the sense of touch, unlike the sense of sight, only allows for point (fragmentary) reading of the expression. • Mathematical expressions are written in a linear manner (sequence of signs), limited to 64 characters.
- There is no possibility of varying the size of characters or using rich mathematical symbolism.

The advancement of knowledge and new technologies meant that ***the International Braille Mathematical Notation for the Blind***, developed in 1955 in Marburg by a team led by Helmut Epheser, required changes. They were introduced under the direction of Helmut Epheser in 1986 in the Braille version, and then in 1992 in the black-print version. In order to simplify the notation of complicated mathematical expressions as much as possible, and at the same time improve their readability, the way they were written was changed by developing a projection technique.

Principles of projection technique

Projection is the transformation of two-dimensional notation into linear notation. Formulas containing symbols replacing multi-level black-print notation (e.g. fractions, indices, exponents, or roots) are called projectors.

There are three types of projectors: simple, complex and detailed.

Simple projectors They

start with a simple exponent sign /, a simple superscript sign [or subscript sign 0, or a simple square root sign 2. These signs are followed by simple expressions containing characters standing in one row, i.e. they do not contain other projectors, fractions (apart from ordinary ones). There can be no empty characters inside simple projectors. Places that, according to the rules of writing must be left blank, please fill in with the ! sign.

Each of these projectors ends:

- through an empty character,
- by the sign closing the simple projector 4, respectively also by 4 or #4, if the simple projector is a component of a complex or detailed projector, c) by another projector sign,
- by a fraction bar or fraction
- termination sign, e) by a closing bracket when the corresponding opening bracket comes before the projector, f) by exponents and indices, which are natural numbers, written after the key sign as reduced numbers.

$$a^{n+m} = a^n a^m$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{a_1 a_2 \dots a_n}{2}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^k)^2 = a^{2k}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{\sqrt{3}}{2} \frac{1}{2} - \sqrt{3}$$

2"b7:_<"`:-2"b) 5

$\ddot{y} + an\frac{1}{2}$

(3 1"b_ *&`0m!* "a?-"d

$f_n(\alpha) Bx$

&e0m41w?_<mw **Compound projectors**

Start with a compound

exponent sign /, a compound superscript sign [or subscript sign 0, or a compound square root sign 2.

They can contain simple projectors and fractions in which there are no blank characters. The action of a compound projector is terminated by: a) a

blank character,

b) the end sign of the composite projector 4, if necessary also by #4 if the projector compound is part of a detailed projector, c) by

another compound or detailed projector sign, d) by a closing

bracket if the corresponding opening bracket comes before the projector sign, e) by a fraction end designator if the fraction bar sign comes before the projector sign.

There must also be no empty spaces in the complex projectors. If necessary, the filler point ! is used. If the ambiguity of the notation is excluded, the sign can be omitted announcing composite projector.

$$\sqrt[\text{and}]{x} = \sqrt[\text{and}]{x}$$

2&w7x_<2w72x

$$un=\sqrt[n]{3+2^n}$$

&t0m_<[m 2"b/m4!* "a/m

$$\frac{x}{\text{and } 2} = \sqrt[\text{and}]{x}$$

&d /w7:_< 2d/w

Detailed Projectors They start

with simple projector designations preceded by the # sign. They can contain any expressions, other projectors, and blank characters. The characters ending with the detail projector #4 must be used at the end.

$$\sqrt[3]{1x+2x}$$

&d#/ [92w!* "` _*"a#4-d/w

$$u = \sqrt[3]{x^2 + x\sqrt{+^2} + 1} 8$$

&t_<[9#2w/:_* 2w/!*" "` _*"g#4

Rule 3 If

the numerator and denominator are natural numbers, then the fraction line can be omitted and the denominator written in the lowered position. This is how it is written:

Common fractions

$$\frac{1}{2} \quad \frac{3}{14} \quad \frac{17}{5} \quad \frac{138}{43}$$

Similarly, it is written:

Mixed numbers

The integer part and fractional part of a mixed number are preceded by **a number sign**.

$$2\frac{3}{4} \quad 4\frac{7}{15} \quad 12\frac{14}{17}$$

$$2\frac{3}{4} \quad 4\frac{7}{15} \quad 12\frac{14}{17}$$

Rule 4

If there are any blank spaces inside the fraction, it is written as follows:

- a) The denominator is written directly after the fraction line (without a space), and in place of the empty spaces characters, the filler character ! is entered. The fraction terminator is omitted, but it is mandatory leaving a blank after the denominator. The numerator is treated in a similar manner.

$$\frac{xy^+}{xy} \quad \text{shortened notation} \quad \text{full record} \quad \frac{xy^+}{xy} \quad \frac{xy^+}{xy}$$

- b) The fraction start sign can be omitted only when the numerator is short. It cannot be missing, when the counter starts with a minus.

$$\frac{p}{n} \quad \frac{p}{n} \quad \frac{p}{n}$$

If the nominative case begins with a character that requires a blank character to be left, (e.g. minus), it is replaced by the ! sign.

$$\frac{pq}{n} \quad \frac{pq}{n} \quad \frac{pq}{n}$$

The notation methods described in points a)-b) can only be used when the numerator and denominator do not contain fractions other than common fractions.

- c) If the counter ends with a character that requires a space after it, a space is left.

between it and the fraction bar there is a filler character !.

$$\frac{(n+!)}{2n}$$

$$\frac{(n+!)}{2n}$$

The factorial sign requires leaving behind there is 5_. In the shortened version before no space between the slash and the fraction bar, hence filling this space with a sign !.

Examples of operations with common fractions

$$\frac{3}{4} + \frac{3}{4} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$$

"b._*"b._<"e._<"b:._<"^":

$$,62 \ddot{y} = \ddot{y} = 2 = \frac{18}{30} = \frac{12}{30} = 2\frac{6}{30} = 2\frac{1}{5},$$

"a+e_,"e+>_<"a" g9(_,"`a9(_<
<"a"e9(_<"a">_<"a+a

The multiplication sign, a dot, can be used with or without a space.²⁷ The notation with a dot without a space is shorter. According to some, using a space improves the readability of the formula.

$$4\frac{3}{4} (21\ 80\ 4)^{=y} \quad \frac{3}{4} 40^{=y} = \frac{19}{4} \frac{4}{10} \frac{19}{10} \quad 91$$

Braille notation with a dot without a space.

"c"b._-1" +a_,"i+g?_<"c"b._-i+c_<
<"`h.\_-c+(\_<"`h+(_<"`+h

$$4\frac{3}{4} (21\ 80\ 4)^{=y} \quad \frac{3}{4} 40^{=y} = \frac{19}{4} \frac{4}{10} \frac{19}{10} \quad 91$$

Braille notation with a dot and a space.

"c"b._-1" +a_,"i+g?_<"c"b._-
-i+c_<"`h.\_-c+(\_<"`h+(_<"`+h

Algebraic fractions

The notation of algebraic fractions follows the rules and regulations for notation of algebraic expressions.²⁸

$$3a + b$$

$$3x + y$$

- Once an alphabetic character is placed in an expression, it remains in force until the end of the expression or until it is canceled by another alphabetic character,
- The use of an alphabetic character is required before the first letter mathematical expression.
- The capital letter sign and the Greek lowercase and uppercase letter signs can be used redundantly.

When the numerator is long, you must put a start fraction sign.

$$\frac{3a+b}{4c}$$

$$\frac{6a+4b}{5ab}$$

$$\ddot{y} = \frac{\ddot{y} \ddot{y} \ddot{y}}{2 \cdot 2 \cdot 2} \pm \dots$$

$$;f_<;\text{'!};a7:_<;7:_*;a7:$$

$$2\frac{2}{3} + \frac{23,61}{1\frac{2}{5} + \frac{1}{2}r}$$

$$"a"a9_*: "b+a\&o!, "\text{'}+ep7\text{'}\text{'}"a>!*"\text{'}:q_$$

$$2\frac{2}{3} + \ddot{y}3p \frac{61}{1\frac{2}{5} + \frac{1}{2}r}$$

$$"a"a9_*"b\&o_,""\text{'}+ep7\text{'}\text{'}"a>!*"\text{'}:q_$$

$$\frac{\frac{2}{3} + \ddot{y}23,61}{1\frac{2}{5}} + \frac{1}{2}r$$

$$:"a9!*"\text{'}b+a\&o!, "\text{'}+ep7\text{'}\text{'}"a>_*"'\text{'}:q$$

$$\frac{\frac{2}{3} 23,61 \ddot{y} q, + r}{1\frac{2}{5}} \frac{1}{2}$$

$$:"a9!*"\text{'}b+a\&o!, "\text{'}+ep!*"\text{'}:q7\text{'}\text{'}"a>$$

$$\frac{\frac{2}{3}xy, 51}{2_{and} + \frac{3}{8}^{mm}}$$

[6]

$$:"a9\&w_,""\text{'}+dx_7_ "ax_* *"b'y;$$

The expression [6] is written in full – using the beginning and end signs of the fraction.

$$:"a9\&w!, "\text{'}+dx7"ax!*"\text{'}b'y_$$

The expression [6] written in shorthand.

Brackets – or shorthand?

In mathematical Braille notation, algebraic expressions can also be written uniquely using brackets. This issue is illustrated in the examples below.

$$\frac{-xy^2}{2} = \frac{-(xy^2)}{2} = \frac{-^+xy^2}{2} = \frac{-^+xy^2}{2}$$

,: &w!, x7: _ <:, 1w_, x?7: _ < <:, w!
*x7: _ < x!, w7:

The above expression can be written uniquely using brackets.

$$\frac{-(xy^2)}{2} = \frac{-(xy^2)}{2} = \frac{(-^+xy^2)}{2} = \frac{(-^+xy^2)}{2}$$

,Z1&w_, x?7:_ <, 1w_, x?7: _ < 1, w_
*x?7: _ < 1x_, w?7:

Entering brackets:

- is an interference with the mathematical source text,
- extends the Braille notation,
- when using multiple parentheses for complex expressions it makes reading difficult.

The use of the multiplication sign requires additional explanation – a dot with or without a space. An empty sign before the multiplication sign terminates the projection. In some expressions this affects the differentiation of the notation, the order of which can also be specified by introducing additional brackets.

$$\frac{2x}{3yz}$$

"a&w7"bx-y

Multiplication sign – a dot in notation without a space.

$$\frac{2x}{3} \cdot yz$$

"a&w7"bx_-y

Multiplication sign – a dot in notation with a space.

$$\frac{yx}{and}$$

&d/wx

$$\frac{x}{hey}$$

&d/w_-x

A blank sign before the multiplication sign terminates a simple projector.

Compound

fractions It is proposed to write these types of fractions as follows: the base fraction in the full form²⁹, and fractions appearing in the numerator or denominator in the short form³⁰.

$$\frac{a + \frac{1 + \frac{b}{ab}}{1 - \frac{ab}{ab}}}{ab} = \frac{a + \frac{1 + \frac{b}{ab}}{1 - \frac{ab}{ab}}}{ab}$$

&`!a7`!,a_<:"`\_\*:a7!\_7\_"\_,,:a7`;

$$\frac{\frac{ab}{34}}{x} = \frac{ab}{34x}$$

::&`79_,:a7._7_;w7w!*x;

$$\frac{xx + \frac{-1}{x}}{x - \frac{11x + \frac{-1}{x}}{x}} = \frac{-(1)(1)xyx\ddot{y} + \ddot{y}1}{y} = \frac{1xyxx\ddot{y} + \ddot{y}}{-1} = \frac{1 + \frac{1}{x}}{-1}$$

:&w_*:w!,"`7x!,"`_7_w_,:w!*""`7 7x!\*""!;\_<::w1x\_,"`?!
 1w_,"`?7x!,,"`_7_:w1x_""?!,1w_*""?7x!*""`;& <::wx!,w!
 w!,"`7x!,"`_7_:wx! *w!,w!,"`7x!\*""`;<x!*""`7x!,"`

$$\frac{22 + \frac{2170}{3622160}}{80} = \frac{4}{5}$$

: "a" a9_*: " + a!, "i + f7" a + e_7 7_: " + a!
 * "i + e7 "i + g_, "b" c >;

$$\frac{1}{234} + \frac{1}{223} = \frac{1}{234}$$

"`:\_\*""`7"a-"b_*""`7"a-"b-"c_

Chapter 29 of the Guide – **Full notation of fractions.**

Chapter 30 of the Guide – **Shortened notation of fractions.**

POWERS AND INDICATORS

Key signs for exponents and right-hand indices

from/ simple exponent sign

from simple subscript sign

from from simple superscript sign In Braille,

the exponent and right-hand side indices are written after the basic symbol, each of them should be preceded by the appropriate key sign. The exponent sign, the indicator sign lower and upper indicator sign start the simple projector.

1. Indexes and exponents that are integers are written after the appropriate key sign with lowered digits without a numerical sign. At the same time, they end the projector.

a^{-3} a^0 a^{12}
 $\&\text{`0},9$   $\&\text{`0}($ $\&\text{`0}+:$

b^{-1} b^4 b^{31}
 $\&a[,+$ $\&\text{and}[.$ $\&a[9+$

x^{-3} 5^4 b^{10}
 $\&w/,9$ $"d/.$ $\&a/+($

$a^{\frac{1}{2}}$ $a^{\frac{12}{21}}$
 $\&\text{`0}++\text{`0}::_,\text{`0}+:\text{`0}:+$

$232 a^{cb}$ $2xy^2\sqrt{\quad}$ $\frac{x^3}{3}$
 $\&\text{`/:}a/9b/:$ "a&w/:2x &w/97"bx

$\ddot{y}^{\frac{1}{3}}$ $\ddot{y}^{-4}$ $\frac{1}{2}$ $8^{\frac{1}{3}}$
 $\ddot{y}^{\frac{1}{3}}$ $\ddot{y}^{\frac{1}{3}}$ $\ddot{y}^{\frac{1}{3}}$ $\ddot{y}^{\frac{1}{3}}$
 $"^9/, "c \&\text{`/}"^:$ "g/,9_<"^7" g/9

2. The exponent or right-hand indicator can be a simple projector. It contains the signs standing in the same row (it does not contain other projectors or fractions other than ordinary).

There can be no blank characters inside the projectors. A simple projector ends with:

- a) nearest empty character

x^{n+9} z^{n-3}
 $\&w/m_*"h \&x/"am!, "b_ -y$

a^{n+2} a^{n+1-5}
 $\&\text{`0}m\_*"a \&\text{`0}m!*"_, "d$

b) the character ending the simple projector 4, through a pair of characters 4, or #4, when the simple projector is part of a more complex projector

xi xj $\cdot 2^{n-3}$
 $\&w0h4-w0i \&x/"am!",b4-y$

$f_n(x)$ $f_{n+1}(x)$ $g_{ij}(y)$
 $\&e0m41w? \&e0m!*"41w? \&f0hi41x?$

c) by another sign starting the next projector

$(an)^k$ $(P2n-1)^m$
 $\&`0m/j \#o0"a\&m!,""/l$

d) by a fraction bar or fraction terminator

$\frac{x^n}{n!}$ $\frac{1}{4 \star n \pm}$
 $\&w/m7m5 "`7"c!*\&w0m!*"``;$

e) by a closing bracket when the corresponding opening bracket is in a different row. $(2 + xn + 5) 2$

$1"a_*\&w0m!*"d?/:$

3. The exponent or index can be a composite projector. A composite projector can contain simple projectors and fractions, inside which there are no empty spaces. It opens with the appropriate key sign preceded by the composite projector sign.

with / composite exponent sign

zz 0z composite subscript sign

with {z composite superscript sign

The composite projector ends

with: a) the nearest blank character,

b) a pair of characters ending the composite projection 4, or #4 when the projector is composite part of the detail projector,

c) by a sign starting the next complex or detailed projector, d)

by a closing bracket, when the corresponding opening bracket is in a different row,

e) by marking the end of a fraction, if the fraction bar sign is before the projector

sign. $f(x)$
 $\&e 0\&m0j 41w?$

PPPP $x_1^4, x_9, \dots, x_{n^2}$

#o0+&+ _#o0.&+ _#o0)&+ _---+& #o 0&m/: 4

 $x_1^{n_1}, x_2^{n_2}, \dots, x_{n_k}^{n_k}$ &w 0m0+ 4&+ _w 0m0: 4&+ _w 0m0. 4! !&+ _---
+ _w#0m 0"a/j#4 $A_1 \bar{a}_1, A_2 \bar{a}_2, \dots, A_k \bar{a}_k^{2^{2k} - 1}$ #`0+ \_<&`+ _#`0: \_<&`/<&+ _#`09\_< <&`/9+&+ _---
+ _#`0&j\_<&`/"a/!
!"oh!,"`4!,"` 4 $\frac{x^2}{\sqrt{2y}}$

:&d /!:w/:7: 472"a;o :&d /!:w/:7: or

4_7_2"a;o;

4. The exponent or index can be a detailed projector. A detailed projector can contain any expressions, other projectors and blanks. It is opened by the appropriate key sign preceded by the detail projector character #. At the end, the characters #4 must be used, which cannot be missed.

from#/from detailed exponent sign

from#0from detailed lower indicator sign

from#{from detailed upper indicator sign

 $x_{n_1}, x_{n_2}, x_{n_4}, \dots, x_{n_k}$ &w /m0+&+ _w /m0:&+ _w /m0.&+& ---+ _&w#/
m 0"a/j#4

Order of right-hand side indices and exponent

When a symbol has more than one simple or compound indices on the right-hand side, they are written one after the other, preceded by the appropriate key sign; moreover, if the basic symbol is an exponent expression, the exponent should be placed at the end after all the indices. (

 x_n and

 $(x)^n$
 $x_{h/q}$
 $(x)^{h/q}$
 x_n
 $(x)^n$
 $x_{h/q}^4$
So-called "stamps"

The term "signs" is understood to mean the designations written at the top or bottom right of the basic symbol or above or below the basic symbol, which are not indicators. Certain geometrical signs, the signs of operations given above, and other signs may be used as signs.

1. Symbols written at the top or bottom to the right of the basic symbol

 $\ddot{y}$
 z prime

 $\ddot{y} z)$
 x bis

 z^* star

 z^*
 z^9 cross

 $\ddot{y}$
 z^9+
 $\ddot{y}$ right arrow

 $-z,$
 $\ddot{y}$ left arrow

 $+z^*$
 $+$ plus

 $^\circ z($
 $^\circ$ minus

 $\neg z_3$
 $\neg$ hook

 $\circ$ circle

2. Symbols written above or below the basic symbol

 $\sim z > ^\circ$
 $\sim$ wave

 $-z^9$
 $^\circ$ line

 $z(\cdot z:$
 $(\cdot$ circle

 $^{\wedge}$ with/

 $^{\wedge}$ peak

 $\cdot$ dot

 $\bullet\bullet$ with::

 $\bullet\bullet$ two dots

The symbols are written after the symbol they refer to. They stand in the same row as the basic symbol. The position of the symbol relative to the base symbol determines the corresponding key symbol between it and the symbol. The key symbols used for the symbols are as follows:

 $\tilde{z}$ upper mark key

 $\underset{\sim}{z}$
 $\underset{\sim}{z}$ bottom mark key

The lower characters must always be written with a key sign, while the upper characters may be omitted if there are no conflicts.

Examples
Full record

 R^+ $##q=^*$

AND*

 $\#`=))$

cŷ

 $\&B;,,$

ŷ

 C $\#b=/$

~

AND

 $\#`= >$  $b^{\hat{y}}$  $\&a=9+$  $A\hat{y}$  $\#`a=9+$

Short form

 R^+ $##q^*$

AND*

 $\#`))$

ŷ

 C $\#b/$

~

AND

 $\#`>$  $b^{\hat{y}}$  $\&a9+$  $A\hat{y}$  $\#`a9+$

In a situation where a symbol has both a symbol and an index or exponent, then in Braille the symbol is usually written immediately after the symbol, before the indicator or exponent.

$$\cdot$$

 x^{n+1}

$$\hat{y}$$

 an

$$\overline{-2}$$

 in

$$\&w:0m!*^{\hat{y}} \&`))0m \&u9/:$$

The reverse notation is also possible, in which the sign is written after the index or exponent. However then it is necessary to show the end of the indicator or exponent using the end of straight line projection sign 4.

$$\hat{y}$$

 $1y$

$$\hat{y}$$

 X
 $+ 1n$

$$\&x0+4) \&w0m!*^{\hat{y}} 4))$$

Horizontal brackets the sign

written above the expression

a sign written under the expression

$$\hat{y} \#9+z \hat{y} ^9+z \hat{y} \# 9z \hat{y} ^$$

long right arrow

$$9z \text{ — } \#9z \text{ — } ^9z \#1z$$

long left arrow

$1z$

long line

$$\text{—}$$

$$\text{—}$$

long bow

a sign written above the expression

 $\overset{\frown}{\#8z}$

a sign written under the expression

 $\underset{\frown}{^8z}$

long lying brace

Bracket characters are always written before the part of the expression that is to be emphasized.

 $\overset{y}{AB}$
 $\overline{CD}$
 $\overbrace{KLM}$
 $\overbrace{AB11C1}$
 $\#9+\#`a \#9\#bc \#1\#jkl \wedge 1\#`0+a0+b0+$

Left hand indicators

Key signs for left-hand indicators full notation short notation

"{z _{z - upper left index character

"0z _0z - lower left index character

1. The abbreviated notation should be used in the records. The full notation of left-hand indicators should be used if the shorthand notation is ambiguous.
2. The left-hand indicator sign precedes the main symbol to which it refers. The left-hand indicator sign begins a simple projector. The indicator usually ends the end-of-projector sign. It can be omitted when the indicator is a number written in lowered digits or when the indicator is followed by another indicator sign.
3. If the left-hand indicator is a complex or detailed projector, we use complex indicator signs or detailed indicator signs (analogous to the right-hand indicators).
We finish the pointer with the appropriate projector end marks.
4. If a symbol has a left upper and lower indicator, they are written one after the other. Each is entered by the appropriate character. There is no rule regarding the order in which the indicators are written.
5. In Braille notation, the root degree is treated as a left upper indicator. It is not necessary to use a terminating indicator sign.
6. The basics of logarithms, which are written in black print in subscript after the logarithm sign, should be written in Braille as upper left hand indicators.

 j_m
 $0\&h\{i4|$

or

 $[\&i0h4|$

or

 $"0\&h\{i4|$
 $\sqrt[3]{8} \neq$
 $\sqrt[n]{x}$
 $\sqrt[n+1]{y}$
 $\{92" g_<" a [\&m2w$
 $[\&m!*"" 2x$
 ${}_2 \log^{8.3}$
 $\{ :5k" g_<" b$

ELEMENTS

Elements are divided into simple, compound, and detailed elements. These names correspond to the types of projectors they are associated with.

from The sign preceding the degree of the root that is higher than the second.31

The degree of the root is treated as the left-hand superscript and is given before the root sign.

Simple square root sign A

simple square root sign starts a simple projection. Therefore, an expression marked with a simple square root sign can only contain characters that are in the same row (it does not contain other projectors, nor fractions other than ordinary ones). There can be no blank characters inside the expression.

$$\begin{array}{ccc} 2z & \text{Simple square root sign} & \\ \sqrt{16} & \sqrt{81}=9 & \sqrt[3]{27} \ 3 \\ 2^{\text{e}} \ 2^{\text{g}} \ _ <^{\text{h}} \ \{92^{\text{af}} \ _ <^{\text{b}} \\ \sqrt{x + \frac{1}{2}} & & 3\sqrt{2x} \\ 2\&w!^{\text{*}}\text{``}: & & \text{''b}2^{\text{''}}\text{a}\&w \end{array}$$

The operation of the simple square root sign is completed by:

a) the nearest blank character,

$$\sqrt{x + y} \qquad \sqrt{x + y + x\sqrt{y}}$$

2&w_ *x 2&w!^{\text{*}}x_ *2w!,x b) a character ending the projector 4 or a pair of characters ending the

projection 4, or #4, when

a simple projector is part of a more complex projector,

$$\begin{array}{ccc} \sqrt{x + y} \ \ddot{e} & & \sqrt{x + y} \ \ddot{y} \ \sqrt{\ddot{y} \ y} \\ 2\&w!^{\text{*}}x4\text{-d/y} & & 2\&\text{in!}^{\text{*}}x4\text{-2in!},x \end{array}$$

$$3\sqrt{2x} \qquad 50\sqrt{x} \ \frac{x\cancel{y}}{x\cancel{y}}$$

$$\text{''b}2^{\text{''}}\text{a}4\&w \ \text{''i+d}2\&w4:w \ _ *x \ _ 7 \ _ w \ _,x;$$

c) the sign starting the next projector,

$$\begin{array}{ccc} \sqrt{a} = a\sqrt{\sqrt{}} & & a\sqrt{2a} \ 3\sqrt{b} \\ 2\&\text{a} \ _ <^2 \ 2a & & \&\text{``}^2\text{''}\text{a}\&\text{``}^2\text{''}\text{b}\&a \end{array}$$

d) a closing bracket when the corresponding opening bracket is in a different row,

$$(a + \sqrt{ab}) \cdot b$$

$$1 \& _ * 2 \` a ? / : _ , a$$

e) a fraction bar or fraction terminator.

$$\frac{\sqrt{2}}{2}, 0,7071$$

$$2 \text{ " } a 7 \text{ " } a _ > > \text{ " } i + f i f \`$$

Compound root sign

Compound root sign is obtained by placing the composite projector sign before the simple root sign. Simple projectors and fractions can be under the root sign. There can be no blank signs.

$$2z \quad \text{Composite square root sign}$$

The end of a composite root is determined according to the rules for the end of a composite projection.

$$\sqrt{2 \sqrt{2}} = 2 \sqrt[4]{2}$$

$$2 \text{ " } a 2 \text{ " } a _ < 2 \text{ " } a _ \{ . 2 \text{ " } a$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$2 \& \` 7 a \_ < 2 \` 7 2 a$$

$$\sqrt{x + y \sqrt{2xy}}$$

$$2 \& w ! * x ! , \text{ " } a 2 w x$$

$$\sqrt{2x + x \sqrt{2y^3}}$$

$$2 \text{ " } a \& w ! * 2 w ! \text{ " } a 4 ! \text{ " } b$$

Detailed root sign The detailed root

sign is obtained by placing the detailed projector sign before the simple root sign. Below the root sign there can be any expressions, other projectors and empty signs.

$$\# 2z \quad \text{Detailed root sign}$$

At the end of the detail root there must be a pair of characters #4, according to the rules of the end of the projector detailed.

$$\sqrt{2x\sqrt{+2}\sqrt{+5}}$$

#2"a&w 2w!*2"a!,w 4_"*d#4

$$\sqrt{x+y}\sqrt{2x}\sqrt{+y}$$

#2&w_*x_,"a2w!*x#4

$$\sqrt{x+y}\sqrt{2x}\sqrt{+y+x}\sqrt{+}$$

#2&w_*x_,"a2w!*x_*2w#4

$$\sqrt{\sqrt{\sqrt{\frac{1}{8}} - \frac{1}{8}} - \frac{1}{8}}$$

#2 22"``4!,"``_,"``#4

Examples of writing expressions using powers, roots and pointers

$$a + b^2 = a^2 + 2ab + b^2$$

(1&`\_a?/:\_<`/:_*"a&`a_*a/:

$$C^k$$

$$\sqrt[n]{k}$$

$$\frac{\text{and } x}{\text{and}}$$

$$\frac{x}{\text{and and}}$$

#b[&j #u[&j0m &`/w7x &` /w7x

$$x_2 = \frac{bb\sqrt{+}\sqrt{42}\sqrt{ac}}{2_{\text{and}}},$$

&w0:_<: ,a_* 2a/!:,"c&`b\_7\_"a-`;

FUNCTIONS

Functions

$$y = f(x) \quad \& x _ < e1w?$$

$$f(x) = 2x \ddot{y} 1 \& e1w? _ < "aw _, " \`$$

$$y = 2x \ddot{y} 1 \& x _ < "aw _, " \`$$

notation using the right arrow symbol $_9 + of32$

$$x \ddot{y} y = 2x \ddot{y} 1 \& w _ 9 + x _ < "aw _, " \`$$

$$x \ddot{y} 2 x \ddot{y} 1 \& w _ 9 + "aw _, " \`$$

$$\ddot{y} (x) x f = 2 + x \ddot{y} 1 \& x \ddot{y} 2 +$$

$$\& e1w? _ < "aw /: _ "*" cw _ "*" e _ <$$

$$< "a1w _, " \` ? /: _ "*" g$$

$$ln = (1.8)$$

$$\#v _ < 1 " \& + _ "g?$$

$$fmax = f(1) = 8$$

$$\& e0 "w _ < e1 " _ ? _ < "g33$$

The expression opposite gives the coordinates of the vertex of the function $f(x)$.

Inverse

function $\ddot{y} 1 f \& \{, +$ inverse function designation

$$f(x) = \sin x \quad \& e1w? _ < 5rw34$$

$$f^{-1}(x) = \arcsin x \quad \& e\{, + 1w? _ < 5 + rw35$$

$$y = \frac{3}{x} \ddot{y} 4 \quad 1 \quad \& x _ < "b.w _, " \& x \{,$$

$$y = \frac{1}{x} \mp 3 \quad \frac{4}{3} \quad + _ < "c9w _ "*" c9$$

32 The arrow pointing to the right is one of the symbols presented in the chapter of the Guide – **Basic geometric symbols**.

33 Symbols **max** "w and **min** "m are given in the chapter of the Guide – **Operations on sets and elements of sets**.

34 The signs of trigonometric functions are given in the chapter of the Guide – **Trigonometric functions**.

35 As above.

Compound function

$$f(g(x))$$

$$u = g(x) = 3x + 2$$

$$f(g(x)) = 3(3x + 2) + 2$$

$$u = \sqrt{\frac{at}{at}}$$

$$f(t) = 2t^7$$

$$f(x) = \sqrt{\frac{3x^2 + 2}{3x^2 + 2}}$$

$$f(g(x)) = 2(3x + 2) + 2$$

Constant

$$\text{const } j \quad \text{constant}$$

$$f(x) = \text{const}$$

$$f(x) = j$$

Signum

$$\text{sgn } r \quad \text{signum}$$

$$\text{sgn } 5 = 1 \quad \text{sgn } (-27)$$

$$f(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

Examples of function

$$\text{notations } f_n(x) = nx + 1$$

$$f_0(x) = 1$$

$$f_{n+1}(x) = (n+1)x + 1$$

$$f_0(x) = 1, f_1(x) = x + 1, f_2(x) = 2x + 1, \dots$$

$$F_{nk}$$

$$(x) \#e 0 \&m0j_1w? \quad \text{or} \quad \#e 0 \&m0j 41w?$$

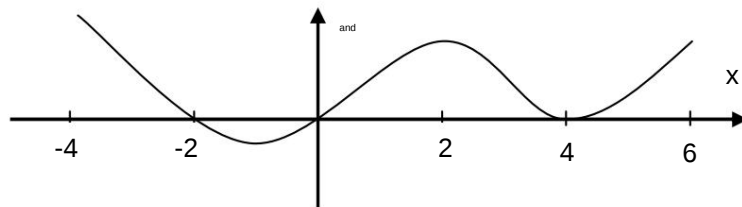
In the first notation, the character is the beginning of the composite projection, the empty character ends the projection. In the second

In the recording, the projection ends with characters 4.

Graphical representation of polynomial graphs

In lessons on polynomials and quadratic functions, especially when solving inequalities, it is necessary to sketch function graphs with accuracy to zeros and the sign of the function value.

Example:



Using Braille characters, such a sketch chart can be represented graphically as follows:

eee_,"a_,"i_eee_"c_eee

The numbers in this line are the zeros of the function. The letters "f" in the notation can be treated as "raised" pluses of the Braille notation, illustrating the fact (and emphasizing it graphically) that in certain intervals limited by zeros, the function takes on positive values, and therefore its graph is above the OX axis. The minus signs correspond to the situation when a fragment of the graph runs below the OX axis, i.e. when the function takes on negative values. It is therefore easy to find out in which places the graph intersects the OX axis (the function changes sign), and in which places we are dealing with a situation in which the graph only "touches" the axis, remaining on the same side of it (the function does not change sign).

Examples:

Quadratic function with two zeros, with the parabola pointing down:

,,,_"b_eee_"e_,,,

Quadratic function with one (double) zero, with the arms of the parabola pointing upward:

eee_,"d_eee

If the entry does not fit on one line, we write it on subsequent lines, dividing it at any space.

,,,_"`\_eee\_,"`:_,,,_"a9 ,,,,_"f_eee_"`_,,,

LARGE CLAMP CONNECTING SEVERAL ROWS

8_ beginning of expressions with a large bracket _;7_ newline character

$$y^{-xx} \text{For } x < 0,$$

$$|x| = y^{-0} \text{ for } x = 0$$

$$y^{-xx} \text{For } x > 0.$$

!k&w^_<8_,w_ck`_&w_8-"i&+_;7_"i&
 Ck`\_&w\_<"i&+\_;7\_w\_ck`_&w_n+"i-

or

!k&w^_<8_,w_ck`_&w_8-"i&+
 ;7_"i_Ck`_&w_<"i&+
 ;7_w_ck`_&w_n+"i-

$$y^{-1} \text{For } x < 0,$$

$$\text{sgn } x = y^{-0} \text{ for } x = 0$$

$$y^{-1} \text{For } x > 0.$$

"r&w_<8_,``_ck`_&w_8-"i&+_;7_"i&
 ck`_&w_<"i&+_;7_``_ck`_&w_n+"i-

or

"r&w_<8_,``_ck`_&w_8-"i&+
 ;7_"i_ck`_&w_<"i&+
 ;7_``_ck`_&w_n+"i-

The newline character is also used to write some expressions where the line feed occurs locally. An example is the Newton symbol.³⁶ In such a case, the newline character is written without spaces.

from; 7fromy a new line character with local action

$$n y$$

$$y^k y$$

1&m;7j?

Another example of using the newline character would be in a combination pattern with repetitions.

$$\overline{C}_n^k = \frac{y^{nk} y^{-1}}{y^k y}$$

#b9{&j0m_<1m!*j!,"`;7j?

EQUATIONS AND SYSTEMS OF EQUATIONS

Equations

$$\begin{aligned}
 2 \times 3(2) &= \tilde{y} & 2) & \quad 10 \\
 3 \times 10 &= \tilde{y} \\
 5 \times 6 &= \tilde{y} & \times & \quad 10 \\
 5 \times 6 &= \tilde{y} & \times & \quad 10 \\
 4 \times &= 4|:4 \\
 \times &= 1
 \end{aligned}$$

Due to the continuity of the operations performed, the previously used alternating convention of notation: black print version – Braille version was abandoned, and the entire solution was first presented in black print and then in Braille.

The Braille notation of the equation solution in the following lines is clear. However, it takes up a lot of space.

$$\begin{aligned}
 &"a-w_*"b1w_,"a?_<w_,"i \\
 &"a&w_*"bw_,"e_<w_,"i \\
 &"d&w_,"e_<w_,"i \\
 &"d&w_,"w_<"e_,"i \\
 &"c&w_<,"c^_."c \\
 &&w_<,"
 \end{aligned}$$

The same solution equation can be written using the logical equivalence sign $_<+z37$, which is a very readable separator for the blind. **The newline character** $_;7_$ can also be used as a separator.

$$\begin{aligned}
 &"a-w_*"b1w_,"a?_<w_,"i_<+ \\
 &"a&w_*"bw_,"e_<w_,"i_<+ \\
 &"d&w_,"e_<w_,"i_<+"d&w_,"w_< <"e_,"i_ \\
 &<+"c&w_<,"c^_."c_<+ &w_<,"
 \end{aligned}$$

The black-print transcription of the above solution seems less legible. 2 3(2) 2 3 6 10

$$\begin{aligned}
 \times \tilde{y} &= \tilde{y} & \times & \quad 10 \\
 \times \times \tilde{y} &= \tilde{y} \tilde{y} \\
 5 \times 10 &= \tilde{y} \\
 = \tilde{y} 6 10 \times & \quad 4|:4 \\
 \times &= 1
 \end{aligned}$$

However, it should be borne in

mind: • Braille reading is dot-based (written by fingers). For this reason, not everything that looks clear in black print is equally clear to a blind person. Many students definitely choose the second way of writing.

• In the second way of writing, despite introducing additional characters, one line was saved. • Using a division of the expression at any point³⁹ would certainly result in saving two poems.

Systems of equations

When writing systems of equations and subsequent solution steps, you can use the large brace and line feed symbols⁴⁰

$$\begin{aligned} xy + 5z &= 3 \\ y + 4x + 2z &= 6 \\ xy + z &= 3 \\ 2x &= 7 \end{aligned}$$

$$\begin{aligned} &\&w_{dx} < b_{bx}; 7, cw_{x} * \\ & * a < h, ew_{ax} \\ & 8_{\&w_{ax} < b}; 7_{aw_{x} < f} \end{aligned}$$

Descriptive and logical connectives can be used to record subsequent steps of the solution.

$$\begin{aligned} &\&w_{dx} < b_{bx} h, cw_{x} * \\ & * a < h, ew_{ax} < + \&w_{ax} < \\ & < b_h aw_{x} < f \end{aligned}$$

The black-print notation of the subsequent steps using descriptive conjunctions is as follows:

$$\begin{aligned} xy + 5z &= 3 \text{ and } 4 \\ 2962 + y + 2z &= 6 \\ = 2i3xy - &= 7 \end{aligned}$$

The readability of the next steps of the solution for sighted people will certainly be debatable. Many blind students, however, choose the linear method of notation.

Signs of determinants and matrices

Lines for the determinant $\begin{vmatrix} \dots \\ \dots \end{vmatrix} \begin{matrix} !kz---z^{\wedge} _ _ kz--- \\ z^{\wedge} _ \end{matrix}$	Determinant module $\begin{vmatrix} \dots \\ \dots \end{vmatrix} \begin{matrix} !kkz---z^{\wedge\wedge} \\ _ kkz---z^{\wedge\wedge} \end{matrix}$
Parentheses for matrices $\begin{matrix} \ddot{y} & \dots & \ddot{y} & \ddot{y} & \dots & \ddot{y} \\ \ddot{y} & \dots & \ddot{y} & \ddot{y} & \dots & \ddot{y} \end{matrix} \begin{matrix} 1z---z? \\ 1z---z? \end{matrix}$	Determinant $\det \begin{matrix} 6cds \end{matrix}$

For the system of equations

$$\ddot{y}xy^{\star} \neq 3$$

$$\ddot{y}^2 x = 7$$

determinant:

$$\text{IN} = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = (1)(-1) - (2)(2) = -5$$

$$\#v_{_} < !k^{_} _ "a^{\wedge}$$

$$\frac{_}{_} k^{_} a_{_}, _{}^{\wedge} _ < _{}^{_} -1, _{}^{_} ?_{_}, "a-$$

$$-"a_{_} < , "d$$

or

$$6cds1^{_} _ "a; 7^{_} a_{_}, _{}^{_} ?_{_} < _{}^{_} -1, _{}^{_} ?_{_}, , "a-"a_{_} < , "d$$

$$\text{In} \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = (1)(-1) - (2)(2) = -5$$

$$!k\#v^{\wedge} _ < !kk^{_} _ "a^{\wedge\wedge}$$

$$\frac{_}{_} kk^{_} a_{_}, _{}^{\wedge\wedge} _ < !k^{_} -1, _{}^{_} ?_{_},$$

$$, "a-"a^{_} < !k, "d^{_} < "d !$$

or

$$k6cds1^{_} _ "a; 7^{_} a_{_}, _{}^{_} ?^{_} _ < !k^{_} -1! , _{}^{_} ?_{_}, "a-"a^{_} < !$$

$$k, "d^{_} < "d$$

matrix representation: $\ddot{y}$

$$\ddot{y}^1 2$$

$$\ddot{y}^2 -1 \ddot{y}$$

$$1^{_} _ "a?$$

$$1^{_} a_{_}, _{}^{_} ?$$

LOGARITHY

function designation

log 5k

base 10 logarithm

ln 5k

natural logarithm

from

the sign preceding

the logarithm base other than **e** or 10 41

The base of the logarithm is treated as the upper left-hand indicator and is given before the logarithm sign.

$$\log 1000 = 3$$

$$\log 16 \bar{4}$$

$$5k'''''_{iii} < "b$$

$$\{ : 5k'' e_{<} < "c$$

$$\ln 2 \neq 2$$

$$\log_{12} 4$$

$$5'k_{&d/} :_{<} < "a \{ '' : 5k''c_{<} < , "a$$

$$\frac{\ln(x+2)}{x-2}$$

$$: 5'k1 \&w_{<} * "a?_{7w_{<}} , "a; 5'k1 \&w_{<} * "a?$$

$$7w!, "a$$

$$\ln \frac{x+2}{x-2}$$

$$5'k_{&w!} * "a7w!, "a$$

or

$$5'k : \&w_{<} * "a_{7w_{<}} , "a;$$

$$\log_{(x_2)} \log x \log_{12} x \text{ and } 2$$

$$\{ \&`5k1w0+ -w0 : ?_{<} < \{ \&`5kw0+_{<} * \\ * \{ \&`5kw0 :$$

$$\log_{xmx_a} \log_{and}$$

$$\{ \&`5kw / l_{<} < l - \{ \&`5kw$$

$$\log_{and} = \frac{1}{\log_{and} x}$$

$$\{ \&w5kx_{<} < '' 7 \{ x5kw$$

Using a blank character after the function name and before an algebraic fraction makes the fraction an argument to the function.

GEOMETRY

Basic Geometric Symbols

⊥	"-	perpendicular
⊥	⊥)"- !	non-perpendicular
∥	kk ⊥!	in parallel
⊥	kk ⊥>	non-parallel
~	⊥)>	is similar
~	⊥<<	it is not similar
≡	⊥)<<	is congruent
≠	68 6'	is not appropriate
∠	6.	corner
⊥	6..	right angle
△	6<	triangle
⊥	6\$	right triangle
□		square
□		rectangle
○		circle
⊙		diameter
6(6) ȳ		prime
z) ȳ z))		bis
#9z		episode
#99z		simple
— #1z		upper arch
— ;1z		lower arch
ȳ 9+z ȳ		arrow pointing right
9z ȳ 9+z		arrow pointing left
		arrow pointing right and left

$AB \parallel CD$ $AA \parallel BB$

#`a_!kk#bc #``)_"-#aa)

 $\angle B = \angle ABC$ ab

68#a_<68#`ab -`_)!kk-a

 $\triangle ABC \sim \triangle A_1B_1C_1$

6.#`ab\_>6.#`0+a0+b0+_>6.#`)a)b)

 $\widehat{ABC}$ $C \parallel AB$ $\overline{AC} \parallel \overline{DB}$ #1#`ba #b\_!d#9#`a #9#`b\_ "-#9#ca #b\_!d#9`a #9#`b_ "-
or #9ca

Vector symbols

The following keywords are used to denote vectors:

 $\vec{}$ #9+z vector pointing right $\vec{}$ # 9z vector pointing left

The vector arrow keyword is the bracket character 42. This means that it includes the entire expression that follows it.

 $\vec{AB} = -\vec{DC}$

#9+#`a_<,#9+#CB

 $\vec{V} = \vec{v}_1 + \vec{v}_2$

#9+#u_<#9+&u0+_ *#9+u0:

 $\text{straight } PP_0 \parallel \vec{at}$

#99#OO0(_!kk#9+&t

 $\vec{y} = \vec{r} \text{ in } u,$

;`_<681#9+&u+_#9+t?

$\vec{AB}$ $\vec{DC}$
Vectors AB and DC are opposite.

A straight line passing through the points
 P and P_0 parallel to the vector $\vec{at}$.

Analytic geometry

Distance of a point from a line

$$|Ax + By + C| = \sqrt{A^2 + B^2} \cdot d$$

$$d = \frac{|Ax + By + C|}{\sqrt{A^2 + B^2}}$$

$$d = \frac{|Ax + By + C|}{\sqrt{A^2 + B^2}}$$

$$d = \frac{|Ax + By + C|}{\sqrt{A^2 + B^2}}$$

or

$$d = \frac{|Ax + By + C|}{\sqrt{A^2 + B^2}}$$

The area of a triangle with the given

vertices (),

B (,),

C (,).

$$S = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$P_{ABC} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$P_{ABC} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$P_{ABC} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$P_{ABC} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$P_{ABC} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

43 The corresponding signs are in the chapter of the Guide – *Signs of determinants and matrices.*

TRIGONOMETRY

Degree measure of an angle

$\overset{\text{full record}}{\circ} = (\overset{\text{shortened notation}}{(' =) })''$
 $\overset{\text{angle degree sign}}{^{\circ}}$
 $=))))$
 $\overset{\text{arc minute sign}}{'} \overset{44}{44}$
 $\overset{\text{arc second sign}}{''} \overset{45}{45}$

$$\ddot{y} = 30^{\circ}$$

;`_<"bi(

$$\ddot{y} = \frac{1}{5}$$

;a_<"`>=(

In the above expression the full degree notation was used because the shortened 1 was mistakenly used

record $;a_<"`>($ this expression would be read as $\ddot{y} = \frac{1}{50}$.

$$19^{\circ} 32' 74''$$

"`h("ab=)"cf=))

Notation of minutes and seconds of arc almost always requires the use of full notation.

Arc measure of

$\overset{\text{radian}}{\text{radian}}$
 $\overset{\text{angle}}{\text{angle}}$
 $6q`c$
or
 $"q$

$$90 = \frac{\ddot{y}}{2} \text{ council}$$

"hi(_<;o7:6q`c

or

"hi(_<;o7:"q

$$\overset{1 \text{ council}}{1} = \frac{180^{\circ}}{\ddot{y}} \frac{180.3^{\circ}}{14159} \frac{57^{\circ}}{5471} \overset{99}{99}$$

"`"q\_<"`gi(7;o_>>"`gi(7
7"b+`c`dh_>>"df("`f=)"cd=))

44 Due to ambiguities, the shortened notation of arc minutes is very rarely used.

45 Due to ambiguities, the shortened notation of arc seconds is very rarely used.

Trigonometric functions

Symbols of trigonometric functions

son 5r sine

something 5b cosine

tg 5s tangent

ctg 57 cotangent

sec 5, secans

cosec 51 cosecant

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

5r"ei(_<2b7"a

$$\sin^2 \tilde{y} + \cos^2 \tilde{y} =$$

5r/,:`\_\*5b/,:`_<"`

Examples of trigonometric functions

$$\frac{\text{something } \tilde{y}}{2}$$

$$\text{something} \left(\frac{\tilde{y}}{2} \right)$$

$$\text{tg} \frac{\tilde{y} \cdot 90 \tilde{y}}{2} \frac{\tilde{y}}{\tilde{y}}$$

5b;`7: 5b1;`7:? 5s1"hi(!,;`7:?

Using a blank character after the function name and before an algebraic fraction makes the fraction an argument to the function.

$$\frac{\tilde{y}}{\text{something } 2}$$

$$\text{tg} \frac{90 \tilde{y}}{2}$$

5b_ ;`7: 5s\_"hi(!,;`7: 5b; ;`7: 5s:"hi(!,;`7:

or

5b; ;`7\_"a; 5s:"hi(\_ ,;`7_"a;

or

$$\text{tg} \frac{\tilde{y}}{2} = \frac{1 - \text{something}}{\sin}$$

5s_ ;`7:\_<:"`!,5b;`75r;`

$$\sin \frac{\tilde{y}}{2} = \sqrt{\frac{1 - \text{something } \tilde{y}}{2}}$$

5r_ ;`7:\_<\*, 2:"`!,5b;`7: 4

$$\text{something} \left(-\frac{\ddot{y}}{3} \right) \text{ something} \frac{\ddot{y}}{3} 180^\circ = \text{something} (60^\circ) = \frac{1}{2}$$

$$5b1,;o79?_<5b1,;;;o79_7_;;o;-$$

$$-"\`gi(?_<5b1,"ei(?_<"`:$$

$$\text{something} \text{ec} 30^\circ = \frac{1}{\text{son } 30} = \frac{1}{-1} = -1$$

$$51"bi(_<"`75r"bi(_<"`7"_: _<"a$$

Inverse trigonometric functions

Inverse trigonometric function symbols

arcsin	5+r	arcsine	arcos	5+b	arc cosine
arctg	5+s	arctangent	arcctg	5+7	arc cotangent
arcsec	5+,	arcus secans	arccosec	5+1	arc cosecant

$$f(x) = \operatorname{tg} x$$

$$\&e1w?_<5sw$$

$$\operatorname{arctan} x$$

$$\&e\{,+41w?_<5\ sw$$

$$\arcsin(30^\circ) = \frac{1}{2}$$

$$\text{For } n = 0, \pm 2, \dots,$$

$$5+r1"bi(_*\&m-"bei(?_<"`:$$

$$\text{ck`}_\&m_\<"i\&+_\&,"`_\&+_\&,"a\&+_\&---$$

MATHEMATICAL LOGIC

Logical

symbols $\dot{\vee}$ or

$\dot{\vee}$ and $\dot{\wedge}$

$\dot{\vee}$ $\dot{\wedge}$ $\sim$ this

$\dot{\vee}$ $\dot{\wedge}$ if and only if

no (it's not true that)

Quantifiers

$\dot{\vee}$ or $\dot{\vee} \dot{\vee}$ or $\dot{\vee} \dot{\vee}$ detailed (there is such ... that)

$\dot{\wedge}$ general (for each ...)

$a \dot{\vee} b$

$\dot{\vee} \dot{\vee} a$

$((p \dot{\vee} q) \dot{\vee} (q \dot{\vee} r)) \dot{\vee} v$

$11 \dot{\vee} \dot{\vee} p \dot{\vee} 1p \dot{\vee} q \dot{\vee} \dot{\vee} u$

$\dot{\vee} x \dot{\vee} 2 \dot{\vee}$

$\dot{\vee} \dot{\vee} w \dot{\vee} \dot{\vee} n \dot{\vee} i$

$\dot{\vee} x x + 1 < 0$

$\dot{\vee} \dot{\vee} w \dot{\vee} \dot{\vee} 8 \dot{\vee} i$

$\sim \dot{\vee} x p(x) \dot{\vee} \dot{\vee} x \sim p(x)$

$9) \dot{\vee} \dot{\vee} w \dot{\vee} 1w \dot{\vee} \dot{\vee} \dot{\vee} w \dot{\vee} 9) \dot{\vee} 1w \dot{\vee}$

PROBABILITY AND COMBINATORICS

Factorial

! 5_ factorial

$$5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$$

"d5_<"`-"a-"b-"c-"d

$$\frac{n!}{k!} = \frac{n!}{k!}$$

1&m;7j?_<:m5_7_j5!1m_,j?5!;

$$c_k = \frac{n!}{k!}$$

#b{&j0m_<1m;7j?

$$\frac{-k}{C_n} = \frac{n!}{n! - k!}$$

#b9{&j0m_<1m!*j!,"`;7m!,"`?_< <1m!*j!,"`;7j?

$$v_k = \frac{n!}{n! - k!}$$

#u{&j0m_<:m5_7_1m_,j?5!;

$$\overline{v_k} = n$$

#u9{&j0m_<m/j

A space should be put after the factorial sign, and if for some reason we need it, to fill in, we use the ! sign.

BOUNDARIES

Symbols used in the notation

of limits

limes

 $\lim_{x \rightarrow \infty} f(x)$

infinity symbol

 $\rightarrow$

"striving for"

The symbol "tends to" is a composite of the filler sign
! and the abbreviated arrow symbol $\rightarrow$.

$$\lim_{x \rightarrow 0} \frac{1}{x} = 0$$

"k0&w!9+"\$ _ " 7w _ <"i

$$\lim_{x \rightarrow 0^+} \frac{1}{x} = y$$

"k0&w!9+"i=* _ " 7w _ <"\$

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 1$$

"k0&m!9+"\$ _ m!* _ " 7m _ <"`

$$\lim_{x \rightarrow 0} 2^x = 1$$

"k0&w!9+"\$ _ "a/,w _ <"i

$$\lim_{x \rightarrow 0} \frac{x(f(x))}{x(f(x))}$$

"k0^c&w!9+"i _ :e1w _ *^c&w? _ ,e1w? _ 7
7 _ ^c&w;

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} (x + 2) = 4$$

"k0&w!9+"a _ :w/:_,"c _ 7 _ w _ , "a; _ <
<"k0&w!9+"a _ 1w _ * "a? _ <"c

$$\lim_{x \rightarrow 0} x(f(x))$$

&d#/"k 0w!9+w0(_e1w?#4

The # sign is the start sign of the detail projector, the # sign
starts composite projection. Compound projector ends
blank sign and detailed #4.

Before expressing $\frac{1}{x}$ a blank character was used

—· It closes the simple projector.

DERIVATIVES

$$x' = y \frac{dy}{dx}$$

derivative

&e)1w?_<cx7cw_<x)

$$\ddot{y}^3$$

partial derivative sign

$$y_{,xg} = \frac{\ddot{y}_{,x} f}{\ddot{y}_x}$$

&f1w+_x?_<3e1w+_x?73w

INTEGRALS

$$\int$$

integral sign

$$^{\wedge}$$

integral dash

$$\int dx x(f) = x(f) + C$$

]&e)1w?_cw_<e1w?_*#b

$$x_2$$

$$\int dx x(f) x(F) x(F) x(F)$$

definite integral

$$x_1$$

] 0&w0+ {w0:_e1w?_cw_< <#e1&w0+? _,#e1&w0:?

$$\int_0^2 x^2 dx = \frac{1}{3} x^3 \Big|_0^2 = \frac{8}{3}$$

]0({:_ "b&w/:_cw_<w/9_^0({:_< <"a/9_,"i/9_<"g

MUTUAL POSITION OF SIGNS

The symbols used in Braille notations and examples of various expressions cited and discussed in the Guide indicate the importance of precise and rule-compliant use of Braille dot characters. • The same Braille character can have multiple meanings.

- The position of the sign in the mathematical text plays an important role.
- Of particular importance is the break in the record (space), referred to as the "blank character".

Due to the proximity of the "empty character", there are 6 groups of 46 characters in Braille mathematical notation. Examples of characters from each group are given below:

Group A – characters written on the left, with or without a space.

$\sqrt{\quad}$	capital Latin letter sign	tg
$2z \#z$		$5s$

Group B – characters written with a space on the left

$+$	$\geq$	γ
$_ *z _ n < z _ < z$		

Group C – characters written without a space on the left side

indicator right side top	γ (prime)	end of the projector
$z\{zz\}$		from4

Group A γ – characters written on the right side with or without a space

const	council	$\triangle$
"j "Q		6.

Group B γ – characters written with a space on the right side

!	{ (large)
$5_ 8_$	

Group C γ – characters written without a space on the right side

print mark	
bold	$>$
from#from n+from	

The immediate proximity of characters from some groups precludes the fulfillment of the above conditions for correct spelling. An example would be characters from group C appearing directly after characters from group B* .

One of the commands requires a space to be kept between two characters, and the other excludes the "blank character".

The definitions of symbols adopted in mathematical notation minimize the number of such cases. However, it is difficult to exclude them completely. In order to resolve such situations, different binding force was given to the individual definitions. • The definitions from

groups C and C̣ are absolutely binding . • The next in importance

are the definitions from groups B and Ḅ . • The last place is occupied by

groups A and Ạ .

In conflict situations, the following rules for the succession of mathematical signs should be applied: 1. Two consecutive signs are written

immediately after each other in two cases: ̣ if the first of them is from group C̣ and the second is any or ̣ the first is any and the second is from group C.

Large opening
clasp sign (group Ḅ)

8_

> (group C̣)

n+z _<z

Right Upper Indicator
Sign (Group C)

from

= (group B)

Together

8{z

n+<z

2. A "blank character" must be left between two consecutive characters when - the first one is from group Ḅ and the second one is from B or A, or - the second one is from group B and the first one is from Ḅ or

Ạ . !

(group Ḅ) 5_

+ (group B)

_*With

Together

5_*z

3. If the first character is from group Ạ and the second from A or vice versa, you can leave a blank between them sign or not, whichever is more convenient to read.

BRAILLE PHYSICAL NOTATION - NOTES

Physical sciences use the tools of mathematics to present their content and analyses. Hence, physical notations in most cases are essentially no different from mathematical notations. In physical dependencies, however, slightly different symbols are encountered, and symbols of physical quantities are often provided with complex indices. The units of physical quantities and their transformations also play an important role. In the following chapters, issues occurring in physical notations will be discussed in an expanded manner. However, these contents constitute an integral whole with mathematical notations. It is assumed that the Reader has familiarized himself with the content of the chapters of the Guide devoted to mathematical notations.

INDICATORS AND "SIGNS" IN PHYSICAL FORMULAE⁴⁷

The use of indicators and "marks" in symbols of physical quantities is very wide. In many cases, they are an inseparable part of the symbol, in others they allow precise determination of the state or course of a process. They emphasize mutual relations between quantities, sometimes indicate certain correlations, opposites, differences. The use of indicators and "marks" in records certainly makes them more orderly, transparent, and therefore more understandable. For these reasons, this part of the Guide will present and discuss selected relationships from various branches of physics, in which additional designations in the form of indicators and "marks" are used. In most cases, right-hand indices are used in the symbols of physical quantities.⁴⁸

z_0z - lower right indicator

$z\{z$ - upper right index

$$v = \sqrt{v_1^2 + v_2^2}$$

$$u < 2u_0 + \dots u_0$$

Indicators clarify the designation of a physical quantity and emphasize the relationships between quantities.

$$\dot{y} = \frac{2 \ln}{T^{1/2}}$$

$$c_{0V} = \frac{Q}{0}$$

$$;k_{<5}k"a7\#s_0": \#b_0(_<P7u_0(\#b_0(_<\#P7\#u_0($$

or

$$U_{sk} = \frac{U_0}{\sqrt{2}}$$

$$x = x \dot{y} t v_0$$

$$\#T_0' \&rj_{<\#t_0(72"a \&w_{<w_0(_,us}$$

$$\frac{s_1}{s_2} = \frac{\dot{y} t_1 \dot{y}^2}{\dot{y} t_2 \dot{y}}$$

$$\&r_0+7r_0:_{<1s_0+7s_0:?:/:$$

⁴⁷ Definitions of right-hand and left-hand indicator signs, "marks", detailed rules for their spelling and the appropriate tables are included in the Guide's chapter – *Powers and pointers*.
 Chapter 48 of the Guide – *Key signs for exponents and right-hand indices*.

$$\frac{1}{R_1} \frac{R_2}{R_1 + R_2}$$

#h0+_<h0(Q0+7q0+!*Q0:
 or #h0+_<#h0(#Q0+7#q0+!*#Q0:
 or #h0+_<:#h0(#Q0+_7_#q0+!*#Q0:;
 $\ddot{y} = \frac{T_2}{T_1}$
 ;4_<#s0+!,S0:7s0+
 or ;4_<#s0+!,#S0:7#s0+
 or ;4_<:#s0+_ ,#S0:_7_#s0+;

$$\ln_{AB} \frac{y}{y} = \frac{y}{r_{AB}} \frac{1}{y}$$

#v0`a\_<,fl&l1"7q0#`_,"7&q0!
 !#and?

The first line of the above Braille pattern is broken after the subscript "B".

The mathematical expression is interrupted at a place where there is no blank character, then as a character the ! character is used for transfer. This character must be repeated at the beginning of the next line.⁴⁹

By using redundant capital letters you can also write:

$$\#v0\#`a_<,\#f\#l\&l1"7q0\#`_,"7 7\&q0\#a?$$

The line was broken at the slash character.

In some notations of physical quantities, so-called "symbols" are used.⁵⁰ The following is a list of the most frequently used "symbols" in physical notations.

$\ddot{y}$	prime	$^{\circ} z$	circle
$z)) + z^*$	bis	$\wedge z / \sim$	dot
$z, - z_9$	plus	$z > * z.$	peak
	minus		wave
	line		star The

symbols are written after the symbol they refer to. They are assigned the same row as the base symbol. The position of the symbol relative to the base symbol is determined by the corresponding key sign between it and the symbol.

⁴⁹ Guide Chapter – *Moving parts of mathematical expressions to the next line.*
⁵⁰ Chapter 50 of the Guide – *So-called "stamps".*

The key characters used for the stamps are as follows:

top mark
 $Z=ZZ;Z$ bottom mark

The lower character key sign is always required. The upper character key sign, if the record is unambiguous, can be omitted.

$$\bar{E} = \frac{1}{2} \bar{v} \bar{m}^{-2}$$

#d9_<"`:'&lu9/:

$\tilde{n}$ $\tilde{y}^y$
 ;m> ;about.

Average values are often indicated by a line above the appropriate symbol.

Arrows in physical records

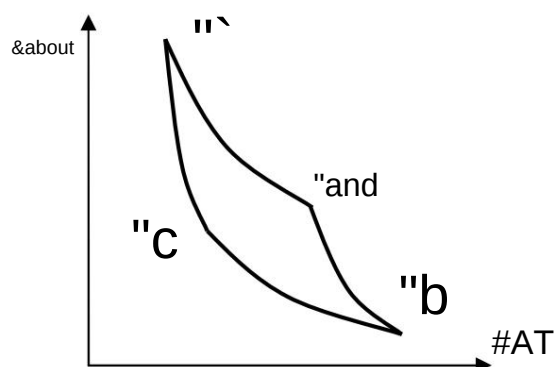
Regular arrows 51

Arrows can be used in descriptions to indicate the direction of changes taking place, as one of the words of an indicator, etc.

The key symbols of such arrows are as follows:

$\tilde{y}$ $_9+$ arrow pointing right
 $\tilde{y}$ $_9$ arrow pointing left
 $\tilde{y}$ $_9+$ arrow pointing right and left

The use of the common arrow symbol is shown below in the description of the Carnot cycle.



1 2 isothermal expansion

"_9+"a_hynsdqlhbemd_qnyoq'4%`, mhd

2 3 adiabatic expansion

"a_9+"b_`ch`a`sxbemd\_qnyoq'4%`, mhd

In processes 3y 4y 1 the gas releases heat Q2

#V_oqnbdr`bg\_"b\_9+"c\_9+"`_f`y
ncc`id_bhdo'1n_#p0:

The regular arrow sign can also be used as one of the words in the indicator symbol. In the case of indicators consisting of several words, after the last word we write 4 end of projection sign or we insert a blank character.

$\ln y_{32} \quad \ddot{y} = \ln y_{14}$

#v0"a!9+"b_<,#v0"c!9+"`

The ! character in the pointer is a filler character. Its use in this case is necessary.

The above expression was mistakenly written without this sign:

#v0"a_9+"b_<,#v0"c_9+"`

according to the rules of Braille notation it would be clearly read as: 3

W2 $\ddot{y} = \ddot{y}W4 \ddot{y}$ 1

It would also be a significant error to ignore the break in the definition of the ordinary arrow symbol.

#v0"a9+"b_<,#v0"c9+"`

The first seven characters are W2 , the next characters are 3. After the equal sign we have W4 ,

and the next characters are 1.

Arrows in vector quantity symbols

The following key symbols are used to denote vector quantities 52

$\ddot{y} \quad \#9+z$ vector pointing right

$\ddot{y} \quad \# 9z$ vector pointing left

$\ddot{y}$
in

$\ddot{y} \ddot{y} \quad \mathbf{F} = \mathbf{has}$

#9+&u

#9+#e_<&l#9+`

The vector arrow keyword written before a quantity symbol is a parenthetical character.⁵³ This means that it includes the entire expression that follows it.

$\ddot{y}$
 $\ddot{y} p$

$\ddot{y} \ddot{y} \quad \mathbf{FAB} = -\mathbf{FBA}$

#9+^c&O #9+#e0#`a_<,#9+#e0#A@

52 Chapter of the Guide – **Vector Notations.**

53 Guide Chapter – **Parentheses written above or below an expression.**

An expression can also be written using the uppercase "redundant" character:

#M0(;9+&#s0"^- #m0(7: ;9+&#s! !0"^-
_#m0(7. ;9+&#s0"^- #m0(7'

The first line of the expression was broken at a place that was not a "blank character". In this case, at the end of the line, place the fourth point !. This character is repeated at the beginning of the new line.

Left-hand indices in physical notation In some physical expressions it is necessary to use left-hand indices.⁵⁶

shortened notation full record

_z "z - upper left index _0z "0z - lower left index

In notations, the abbreviated notation is generally sufficient. The full notation of left-hand indicators should be used when it is necessary to avoid ambiguity in the notation.

²¹⁵₈₄ After $\ddot{y}\ddot{y} + \ddot{y}$ ²¹¹₈₂ Pb ⁴₂ He $\overline{7.4}\text{MeV}$

{: + > 0'. #On ;9+; _{: ++ 0': #Oa_ * *{. 0: #Gd_ * "f + c6 #ld #U In the above reaction, the keyword of the chemical element symbol # was used

symbol of a chemical element constitutes a single entity and should not be treated as an algebraic expression. A lower-case Latin letter is not used before the second letter in two-letter element symbols.

The word symbol of the unit is preceded by the name sign ⁵⁸ 6. In the notation of units, other rules also apply to the use of alphabetic characters. Within the boundaries of a symbol, each capital letter character refers only to the letter it precedes. ⁵⁹

¹⁴₆ C $\ddot{y} + \ddot{y}$ ¹⁴₇ N ⁰₁ $\ddot{y} \ddot{y} + \ddot{y}$ ⁰₀ $\sim$

{ + . 0 * # b _ 9 + { + . 0 < # m _ * { (0 , + ; a _ * * { (0 (; m >

_{e.g.} $\ddot{y}\ddot{y} \sim \ddot{y}\ddot{y}$ ¹₁ $\ddot{y}_1^0 \ddot{y} + \ddot{y}_0^0 \sim$

{ + 0 (& m _ ; 9 + ; a , _ { + 0 + & o _ * { (0 , + ! ! ; a _ * { (0 (; m >

The first line of the expression is broken before the $\ddot{y}$ particle symbol, which starts the second line.

The break occurred in a place where there is no empty square. The ! is used as a transfer sign. This sign starts the next line.

⁵⁵ Guide Chapter – *Moving parts of mathematical expressions to the next line.*

⁵⁶ Guide Chapter – *Left-hand indicators.*

⁵⁷ Chapter of the Guide – *Symbols of elements.*

⁵⁸ Chapter of the Guide – *Units and actions on units.*

Chapter 59 of the Guide – *Word symbols, designation sign* (rule 2).

⁶⁰ Tutorial Chapter – *Moving parts of mathematical expressions to the next line.*

EXAMPLES OF RECORDING DIFFERENT PHYSICAL FORMULAE

Selected Issues of Braille Transcription Braille

transcription allows for a certain amount of freedom in the notation. However, whenever possible, the notation that is the shortest should be used. Below are examples of different correct notations of the same expressions in black print. The titles of the chapters in the Guide, which discuss in detail the issues of using appropriate abbreviations, are given in the boxes.

$$v_1 \quad v_1$$

$$\&u0^{\text{v1}} \quad \&u0^{+}$$

Chapter – **Signs of keys for exponents and right-hand indices** (rule 1).

$$v_0^2 \quad v_0^2$$

$$\&u0^{\text{i/\"a}} \&u0^{(/:}$$

$$\frac{R}{2} \quad \frac{R}{2}$$

$$\#Q7^{\text{\"a}} \quad \#Q7^{:}$$

Chapter – **Short form of fractions** (rule 2).

$$\frac{2 \sqrt{m}}{2} \quad \frac{2 \sqrt{m}}{2}$$

$$\&lu=9^{/:7:} \&lu9^{/:7:}$$

Chapter – **So-called “stamps”**.

$$\frac{T_1 - T_2}{T_1}$$

$$:\#S0+_,S0:7_s0+;$$

$$:\#S0+_,\#S0:7_\#s0+;$$

or

$$\frac{T_1 - T_2}{T_1}$$

$$\#S0+!,S0:7s0+$$

$$\#S0+!,\#S0:7\#s0+$$

Chapter – **Short form of fractions** (rule 4).

Some formulas can be written using simple square root signs.

$$= 2gh\sqrt{vk}$$

$$\&u0j_{<2^{\text{\"a}}}\&fg$$

Chapter – **Simple square root sign**.

However, a similar pattern requires the use of the compound projector sign.

$$t = \sqrt{\frac{2 \text{ hours}}{g}}$$

&s_< 2"a&g7f

Chapter – *Composite square root sign.*

Complex patterns require more detailed explanation.

$$t = \frac{\sqrt{gh v_0^2 \ddot{v}_0}}{g}$$

The above expression in ordinary notation using the start and end fraction signs will have the following form:
character.61

&s_<: 2u0(/:!"a&fg_,u0(_7_f;

In ordinary notation, the numerator of a fraction is enclosed between the fraction start sign and the fraction bar sign. The signs of operations between individual terms of the numerator and denominator are preceded by "empty signs". The root sign precedes the complex projection sign. The operation of the complex root sign is interrupted by an empty sign.

The shortened form of the above expression will be as follows:

&s_<: 2u0(/:!"a&fg_,u0(_7f

In short notation, the fraction bar is in row zero, and the numerator and denominator of the fraction are in row one. The square root sign precedes the complex projection sign. The square root operation ends with an empty sign. There is no space after the fraction bar, and no end-of-fraction sign after the short denominator. In this example, the numerator is too long to omit the start-of-fraction sign.

$$h_{\max} = \frac{v_{0y}^2}{2g} = \frac{2v_0 \sin^2 \gamma}{2g}$$

&g0"w_<u0"ix/:7"a&f_<:"au0(! !5r/;:`7"a&f

In the above Braille notation:

62

γ the abbreviation of the word symbol **max** was used - "w.

γ the fraction bar ends the simple exponent projector.63

Chapter 61 of the Guide – *Full notation of fractions.*

62 Guide Chapter – *Other symbols used in set algebra.*

63 Chapter of the Guide – *Key signs for exponents and right-hand indices*, rule 2.

The use of the multiplication sign between two fractions is explained by the following example:

$$F = \frac{1}{4 \ddot{y}_0} \cdot \frac{q q_0}{r^2}$$

full record

#e_<:"`_7_"c;o;d0(;:-&pp0(_7
7_q/;:

shortened notation

#e_<:"`7"c;o;d0(_-&pp0(7q/:

When writing a shortened product of fractions, the multiplication sign with a space is required.

_-. A "blank character" indicates the end of a fraction. The notation uses an excess of a lowercase Greek letter before $\ddot{y}$.

Examples of expressions using complex key characters

Below are examples of patterns that require the use of compound projections in Braille transcription.

It begins with the sign . The signs 4 end this projection.

$$r = \sqrt{\frac{10N}{\ddot{y} N_0}}$$

&q_< 2"i#m7;o#M0(

$$\tau_2 \ddot{y} = 2 \sqrt{\frac{I}{g \ddot{a}}}$$

#s0:_<"a;o 2&k7f!*`

$$U_0 = U_{R1} + U_{R2}$$

#T0(_<T 0Q0+_ *T 0Q0:

or

#T0(_<#T0#Q0+_ *#T0#Q0:

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

&l_<:l0(_7_ 2"i!,1u7b?/;:

$$N \ddot{y} e_0^{-\frac{\ln 2}{T \frac{1}{2}}}$$

#m_<#m0(&d /,5'k"a7#s0"i: 4

UNITS AND OPERATIONS ON UNITS

Word symbols, title sign In the records of units, a key sign is used for word symbols, in the case of units called a title sign.

6z - keyword of word symbols, name sign

Basic SI units

Size	Unit Name	Symbol	Braille notation
length	meter	m	6l
time	second	s	6r
mass	kilogram	kg	6jf
current	ampere	AND	6#@
temperature	Kelvin	K	6#J
light	candle	cont	6bc
amount of matter	moth	moth	6lnk

Supplementary SI units

Size	Unit Name	Symbol	Braille notation
flat angle	radian	council	6q`c or "Q
solid angle	steradian	wed	"q/:

Derived SI units

Size	Unit Name	Symbol	Braille notation
surface area	square meter	m2	6l/:
volume	cubic meter	m3	6l/9
speed	meter per second	m/s	6l7r
acceleration	meter per second squared	m/s2	6l7r/:
force	newton	N	6#m
pressure	Pascal	Bye	6#o`

Size	Unit Name	Symbol	Braille notation
work, energy	joule	J	6#i
power	watt	IN	6#v
frequency	hertz	Hz	6#gy
density	kilogram per meter ³	kg/m ³	6jf7l/9
electric charge	coulomb	C	6#b
field intensity electric	volt per meter	V/m	6#u7l
voltage	volt	V	6#u
electrical capacity farad		F	6#e
electrical resistance	ohm	Oh	6^v
conductivity	siemens	S	6#r
Electric			
magnetic flux weber		Inb	6#va
tesla magnetic induction		T	6#s
inductance	henry	H	6#h
luminous flux	lumen	lm	6kl
lighting intensity	lux	lx	6kw
activity of a radioactive body	becquerel	Bq	6#ap
ability to concentrate	dioptr	D	6#c

SI unit prefixes

Prefix	Multiplier	Abbreviation	Braille notation
atto	10 ⁻¹⁸	and	6`
femto	10 ⁻¹⁵	f	6e
pico	10 ⁻¹²	p	6o
Nano	10 ⁻⁹	n	6m
Micro	10 ⁻⁶	μ	6;l
milli	10 ⁻³	m	6l
centi	10 ⁻²	c	6b
deci	10 ⁻¹	d	6c
Deca	10 ¹	yes	6c`
hectare	10 ²	h	6g
kilo	10 ³	k	6j
mega	10 ⁶	M	6#l
giga	10 ⁹	G	6#f
tera	10 ¹²	T	6#s
peta	10 ¹⁵	P	6#o
exa	10 ¹⁸	E	6#d

Examples

$$1\text{GW} = 10^9\text{W}$$

$$1\text{kWh} = 3,600,000\text{Ws}$$

$$1\text{nm} = 10^{-9}\text{m}$$

$$1\mu\text{m} = 10^{-6}\text{m}$$

$$1\text{k}\ddot{\text{y}} = 10^3\ddot{\text{y}}$$

Frequently used non-SI units

Size	Unit Name	Symbol	Braille notation
pressure	bar	b	6a
	physical atmosphere	atm	6`sI
time	time	h	6g
	minute	Minutes	6lhm
length	angstrom	Å	6Z
	nautical mile	NM	6#m#l
mass	gram	g	6f
power	mechanical horse	KM	6#j#l
sound intensity	decibel	dB	6c#a
volume	liter	l	6k
surface area	are	and	6`
	hectare	ha	6g`
speed	kilometer per hour	km/h	6jl7g 6v
	nautical mile per hour w. (knot)		
temperature	degree Celsius	°C	(#b or =(#b
	degree Fahrenheit °F		(#e or =(#e

Examples:

$$1 \text{ l} = 1 \text{ dm}^3 = 0.001 \text{ m}^3$$

$$1 \text{ km}^3 < 1 \text{ km}^3 / 9 < 1 \text{ km}^3 / 9$$

$$0^\circ\text{C} = 273.15\text{K}$$

$$1 \text{ km}^3 < 1 \text{ km}^3 / 9 < 1 \text{ km}^3 / 9$$

$$0^\circ\text{C} = 32^\circ\text{F}$$

$$1 \text{ km}^3 < 1 \text{ km}^3 / 9 < 1 \text{ km}^3 / 9$$

$$100^\circ\text{C} = 32^\circ\text{F}$$

$$1 \text{ km}^3 < 1 \text{ km}^3 / 9 < 1 \text{ km}^3 / 9$$

$$3^\circ\text{C} \approx 0.75^\circ\text{C}$$

$$1 \text{ km}^3 < 1 \text{ km}^3 / 9 < 1 \text{ km}^3 / 9$$

$$2 \frac{2}{5}^\circ\text{C}$$

$$1 \text{ km}^3 < 1 \text{ km}^3 / 9 < 1 \text{ km}^3 / 9$$

The second designation °C is used in the records to avoid ambiguity.

$$1 \text{ Å} = 10^{-10} \text{ m}$$

$$1 \text{ km}^3 < 1 \text{ km}^3 / 9 < 1 \text{ km}^3 / 9$$

$$1 \text{ w.} = 1 \text{ NM/h} = 1852 \text{ m/h}$$

$$1 \text{ km}^3 < 1 \text{ km}^3 / 9 < 1 \text{ km}^3 / 9$$

$$1 \text{ dB} = 10 \text{ B}$$

$$1 \text{ km}^3 < 1 \text{ km}^3 / 9 < 1 \text{ km}^3 / 9$$

$$1 \text{ ha} = 10,000 \text{ m}^2$$

$$1 \text{ km}^3 < 1 \text{ km}^3 / 9 < 1 \text{ km}^3 / 9$$

Rules for using the name sign 1. Word symbols preceded

by the key sign 6z

are taken from the black print without any alteration.

Full name	Word symbol	Braille notation	Additional abbreviations for frequently used symbols
radian	council	6q`c	"q
time	h	6g	
hectare	ha	6g`	
meter	m	6l	
candle	cont	6bc	
ampere	AND	6#`	

2. Within the boundaries of the symbol, each Greek letter or capital Latin letter sign refers only to the letter it precedes. When a word symbol begins with a lowercase Latin letter, the alphabetic sign is omitted, i.e., no alphabetic sign is written in such a case.

5kg 2 µm
 "d6jf "a6;l&l
 2Ah 3kWh
 "a6#`g g"b6j#vg

The rules given in rule 2 for word symbols differ from the rules for using alphabetic characters in algebraic expressions (Guide Chapter – **Alphabet Characters**).

3. The word symbol together with the key sign considered as a whole is of type ABÿ 64. This means that an empty box may or may not be left before it, but a space must be left after it.

record without space record with space

5 min "d6lhm "d_6lhm
 10 nF ""i6m#e ""i_6m#e

More often, the notation without a space is used.

Very often the names of physical units are composed of several word symbols. In such a case, instead of write the titular sign before each symbol, it is enough to write the double titular sign 66z before the whole name. Inside the name written in this way there is no need to leave spaces.

J = Nÿm
 6#i_<6#m-6l_<66#MI

$$\text{kg}^{\circ}\text{J} \quad \frac{\text{m}^2}{\text{s}} \cdot \text{m} \quad \frac{\text{kg m}^2}{\text{s}^2}$$
 6#i_<6jf-6l7r/:-6l_< <66jf-l/:-7r/:

4. The name sign is not used for currency names and abbreviations. The name of the currency or its abbreviation should be the number separate with a

break. 5 zlotys

5 PLN

5 PLN

"d_y1nsxbg "d_y'1 "d_#O#K#M

3zŷ 50gr

3.50 PLN

"b_y1_"di_fq "b+di_y1

5. The units written in full are not preceded by the name sign. 3 kilograms

four seconds

bysdqx_rdjtmcx "b_jhknfq`lx

1 hour 30 minutes

2 volts

"`\_fncyhm`_"bi_lhmts "a_vnksx

Main Units - Examples

Work

J = Wŷs

6#i_<6#v-6r

or

6#i_<66#vr

force

$N = \text{kg} \cdot \frac{\text{m}}{\text{s}^2} = \frac{\text{kgm}}{\text{s}^2}$

6#M_<6jf_-6l7r/:_<6jfl7r/:

or

6#M_<66jf-l7r/:_<6jfl7r/:

The double titer 66 can be used in expressions involving operations on units.

Temperature units The

basic unit of temperature in the SI system is **the kelvin** [K] – 6#j. However, the following is very often used: traditional unit **degree Celsius** [°C].

°C from(#B or with=(#B degree Celsius

20°C

36.6°C

$2\frac{2}{5}^{\circ}\text{C}$

"ai(#B "be+e(#B "a"a>=(#b

Units in physical formulas and expressions

Example

Calculate the capacitance of a flat capacitor whose plates have the shape of a square with side $a = 10$ cm, if the distance between them $d = 0.2$ mm.

Data:

$$a = 10 \text{ cm} = 1.0 \text{ m}$$

$$d = 0.2 \text{ mm} = 2 \cdot 10^{-4} \text{ m}$$

$$\epsilon_0 = 8.85 \cdot 10^{-12} \frac{\text{F}}{\text{m}}$$

Braille transcription of the written data and further solution will be as follows:

#c`md9

&`\_<`i6bl<`i+`6l

&c_<`i+a6ll<`a-`i/,.6l

;d0(<`g+gd-`i/,+:6#e7l

$$C = \frac{\epsilon_0 S}{d} = \frac{8.85 \cdot 10^{-12} \frac{\text{F}}{\text{m}} \cdot 10 \text{ m}^2}{2 \cdot 10^{-4} \text{ m}} = 44.0 \cdot 10^{-9} \text{ F} = 44 \text{ nF}$$

#b_<;d0(#R7&c_<:"g+gd-`i/,+;!`

!6#e7l-`i/,+:6l/:\_7\_`a-`i/,..!

!6l;_>>`i+cc-`i/,)6#E_<`i+cc! !6m#E

The next three lines in the above Braille notation were broken at a non-character location.

action, nor an empty character. That is why we put a ! character at the end of the line and repeat this character at the beginning of the next line.⁶⁵

In Braille, operations on numbers can be separated from operations on units. Such a notation seems to be more legible for the blind.

$$C = \frac{\epsilon_0 S}{d} = \frac{8.85 \cdot 10^{-12} \cdot 10^{-2} \cdot \frac{\text{F}}{\text{m}} \cdot \text{m}^2}{2 \cdot 10^{-4} \text{ m}} = 44.0 \cdot 10^{-9} \text{ F} = 44 \text{ nF}$$

#b_<;d0(#R7&c_<:"g+gd-`i/,+:-

-`i/,:\_7\_`a-`i/,,:-66:#e7l\_- -l/:\_7\_l;\_>>`i+cc-`i/,)6#E_<

<`i+cc6m#E

⁶⁵ Guide Chapter – *Moving parts of mathematical expressions to the next line.*

BRaille CHEMICAL NOTATION - NOTES

Braille chemical notation refers to mathematical notation. Chemical symbols and formulas, as well as reaction records, in their essence express completely different content from mathematical expressions. Two-letter symbols of elements constitute a whole and certainly cannot be treated as mathematical expressions.

The notation of chemical reactions is a representation of processes that are difficult to compare with transformations of mathematical dependencies. Therefore, in chemical notations, one can only speak of a certain similarity with mathematical notation. The key signs used in chemical notation and the applicable rules of notation are precisely specified and defined in the subsequent chapters of the Guide.

BASIC SYMBOLS AND SIGNS USED IN CHEMISTRY

The notation shown here uses the empty character to indicate the continuation of the record without a blank character and —.

Element symbols

The element symbol is preceded by *the element key sign*:

#z - root keyword⁶⁶

H	Cl	Fe	S	Mn
---	----	----	---	----

#g #Bk #ed #r #Lm

Molecular formulas

In molecular formulas, the symbol of each element is preceded by *the element keyword*.

HCl	CaO	MgS	CuO
-----	-----	-----	-----

#g#Bk #B`#N #lf#R #Bt#N

If all the symbols of the elements in a molecule of a chemical compound are single-letter, the molecular formula can be written in a shortened way by omitting the symbols of the key elements. The molecular formula should be however, precede it with the key character ⁶⁷ ^ .

^z - the keyword of the molecule with single-letter symbols of the constituent elements

	shortened notation	full record
KOH	^jng	#j#n#g
HCOOH	^gbnng	#g#b#n#n#g
HCN	^gbm	#g#b#m

⁶⁶ In mathematical and literary notation, it is *the key sign of a capital letter* of the Latin alphabet.

⁶⁷ Based on the findings of the International Braille Conference held in Moscow in 1975: *A System of Braille Notation of Mathematics, Physics, Astronomy and Chemistry*, Moscow 1975 [4].

Stoichiometric indices Stoichiometric

indices in the formula of a molecule are written immediately after the symbols of the elements in reduced notation (fifth series characters).

CaSO₄Fe₂O₃MnO₂

#b`#r#n. #ed:#N9 #lm#n:

If the molecular formula contains only elements with single-letter symbols, one can use short notation:

	shortened notation	full record
H ₂ O	^g:N #G:#n	
H ₂ SO ₄	^g:Rn. #g:#R#n.	

Round brackets

Round brackets in molecular formulas are used as in mathematical notations. 68

1z - open round bracket z? - closed

round bracket

Ca(OH)₂Al₂(SO₄)₃

#b`1#n#g?: #`k:1#r#n.?9

Stoichiometric coefficients Stoichiometric

coefficients in chemical reaction equations are written with *the sign of the number* 69 without a space between the coefficient and the formula or symbol.

"z - numeric sign

2H₂3H₂O2Al₂O₃

"a#g: "b#g:#n "a#@k:#N9

You can use the ^ sign before the pattern.

3H₂O2P₄O₁₀

"b^g:n "a^o.n+(

Arrow symbols in chemical notations

$\xrightarrow{\text{left}}$	- reaction going to the left
$\xrightarrow{\text{right}}$	- reaction proceeding to the right
$\rightleftharpoons$	- reversible reaction
$\xrightarrow{\text{gas}}$	- gas leakage
$\xrightarrow{\text{precipitate}}$	- precipitation of precipitate

$\xrightarrow{\text{expression}}$

$\xrightarrow{\text{expression}}$; $\xrightarrow{\text{left}}$ - left arrow written under the expression

$\xrightarrow{\text{expression}}$; $\xrightarrow{\text{right}}$ - left and right arrow written under the expression

Catalysed reaction

In the record of a reaction involving a catalyst or requiring the determination of reaction conditions reaction uses a "long arrow written under the expression". The expression is preceded by the & character and ends with the - character. After the expression is written, an empty character is valid.

$6 \text{H}_2\text{O} + 6 \text{CO}_2 \xrightarrow{\text{C}_6\text{H}_{12}\text{O}_6} 6 \text{O}_2$

"e^g:n_*"e^bn:_;9+&dm-_^b*g+:n*&
*"e#n:_^9`

In the above example, when moving part of the record to the next line, & is used as with the rules for transferring mathematical expressions.

The issue of moving parts of expressions to the next line is discussed in detail in the chapter of the Guide - ***Moving parts of mathematical expressions to the next line.***

Indicators

In chemical notations (electron configuration, valence, ions), left- and right-handed indicators, both upper and lower, are used. These indicators have their key signs in Braille notation.

Keyword symbols of right-hand indicators

$\text{z}\{\text{z}$ - upper right index $\text{z}\}\text{z}$ - lower right index

Left-hand indicator keywords short notation

regular notation

$\text{z}\{\text{z}$ - left upper pointer

$\text{z}\}\text{z}$ - lower left index

In notations, the shorthand notation is generally sufficient. The ordinary notation for left-hand pointers should be used when ambiguity must be avoided.

70 The arrow symbol is also used in physical formulas; chapter of the Guide – ***Arrow symbols in physical notations.***

71 Chapter of the Guide – ***Powers and pointers.***

Ions

Ion charges are given by the right superscript sign.

{, - negative ion {* - positive

ion (shortened notation) {!* - positive ion (full notation)

The full positive ion notation should be used in assays where ambiguity in the reading could occur.

On+ Cl⁻ SO₄²⁻ Ca⁺⁺

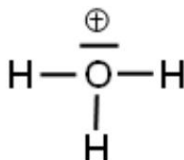
#M`[\* #bk[, #r#n.[,, #B`[**

Hydronium ion

H₃O⁺

^g9n[*

#g + #n + #g
J
#g



Two-, three- and more positive ions and two-, three- and more negative ions can also be written by giving the number of charges immediately after the right-hand superscript sign.

SO₄²⁻ Ca²⁺

#r#n.[!a, #B`[!a!*

In the Ca²⁺ ion notation, the full ion notation is used. In case of mistaken use of the shortened notation, the formula

#B`[!a* would be read as: Ca ^{$\frac{2}{6}$}

Hydrates

In hydrated molecules, **a multiplication sign** is inserted between the formulas of the anhydrous and water molecules. dot without space molecules. dot with space

ZZ

_z - multiplication sign73

Al₂O₃ • H₂O

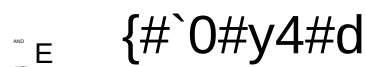
#`k:#n9-#g:#n

or

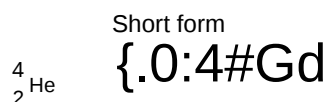
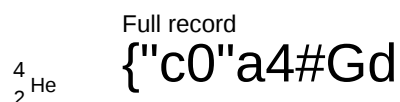
#`k:#n9_#g:#n

The structure of the atom

In the periodic table, each element is described by its atomic number Z (the number of protons) and mass number A, placed before the element symbol as left-hand side markers: A – as a superscript, Z – as bottom indicator 74 ·



Indicators can be written in full or short form. In full form, numbers are written using **a numeral sign**, in short form – the numeral sign is omitted and the number is written at a lower level (fifth series Braille characters).



The projector termination sign 4 should be placed before the root symbol

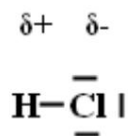
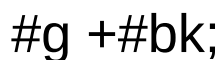
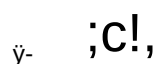
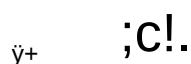
Electronegativity of elements The

difference in electronegativity of combining atoms should be written in accordance with the Greek symbol delta, giving in the right-hand subscript the formula of the compound to which it relates.

$\delta E \text{ NaCl} = 3.0 - 0.9$



Partial positive and negative charges should be represented by the lower case Greek letter delta with a + or – sign (δ^+ or δ^-). A separator sign ! should be placed between the letter and the plus or minus.



The arrangement of valence electrons in an atom

- to the left of the symbol
- on the right side of the symbol
- from the top of the symbol
- from the bottom of the symbol

#b

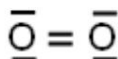


Non-bonding electron pairs

- ; - on the left side
- ; - from the right ,
- from above
- b - from below

$$\begin{array}{c} ' \\ \#n \\ B \end{array}$$


Bonding electron pair arrangement

$$\text{---} \#n \#j \#n$$


Chemical bonds

Ionic bond Each

ion that forms a bond is written in square brackets with the sign of the ion and its value outside the brackets.

[Na]⁺ [Cl]⁻

$\text{Z}^{\#m} \backslash [*_Z^{\#bk} \backslash,$

Covalent bond – notation analogous to the arrangement of bonding electron pairs.

H-H

$\#g + \#g$

PH₃

$\#g$

$_j$

$\#o + \#g _j$

$\#g$

In the case of a **polar covalent bond**, the symbol of the more electronegative element is given after the molecular formula, preceded by the sign $\ddot{-}$.

$\overset{\cdot\cdot}{\underset{\cdot\cdot}{\text{O}}} = \text{C} = \overset{\cdot\cdot}{\underset{\cdot\cdot}{\text{O}}}$

$\#n \#j \#b \#j \#n ; c!, \#n$

H₂O |
 Oh | /\

HH

$_ ; \#n ;$
 $_ _ [0$

Coordination bond – In the molecular formula, all covalent bonds are written. Then, after the blank space, the symbols of the elements forming the coordination bond (donor-acceptor) are given.

An arrow pointing towards the acceptor is placed between the element symbols.

#g + #g



,

#b#j#n_#b 9#n

Electron

Configuration The

number of protons

characteristic of a given element is written as a left-hand subscript before the element symbol. The numbers of electrons in individual orbitals are given as right-hand superscripts after the letter designating the orbital. After the element symbol, before the colon, the sign

separator &. In the absence of the separator, the record can be read as 11Na3 .

11 On :

1 sec 2 2 sec 2 6 3 sec 1 p

0"``#M`&9\_``&r{:_ "a&r{:&o{*&
"b&r{+

19K :

1 sec 2 2 sec 2 6 3 sec 2 6 4 sec 1

0"``h#J&9\_``&r{:_ "a&r{:&o{!
!*_"b&r{:&o{*_"c&r{+

If a mathematical expression is interrupted at a place where there is no empty square, then as a character the ! character is used for transfer . This character must be repeated at the beginning of the next line.

Electron notation In

the electron notation, a dash or minus "sign" can be used⁷⁵ .

—
and

&d9

e3

"b&d9

record with a dash "mark"

-
and

&d, -

3e

"b&d,

record with a minus sign

Valence

- In the notation of valence, Roman numerals are written according to mathematical notation.⁷⁶
- The valence is given after the element symbol as a right-hand superscript.
- In the formula of a molecule, the symbols of subsequent elements are preceded by the projection termination sign 4.

AlIII

CaIIIOII

#@k{#hhh #b`{#hh4#N{#hh

Chapter 75 of the Guide – *So-called* "stamps".

Chapter 76 of the Guide – *Roman numeral notation*.

Oxidation states of elements Roman

numerals are used to record the oxidation states of elements, which are given after the element symbol. in round brackets. Negative oxidation states are given by adding a minus sign before the Roman numeral. If a molecular formula has a stoichiometric coefficient, the oxidation state is written after the coefficient.

Roman numeral notation

IV -II

SO₂

#r1#HU?#n:1,#HH?

For free elements, their oxidation state should be written as an Arabic zero.

Full record

Short form

N₂ #M:1"i?N₂ #M:1(?)

Full record

Short form

0 #n:1"and?

About 2

0 #n:1(?)

About 2

CHEMICAL REACTIONS
Salt formation
Molecular notation
Fe + 2HCl → FeCl₂ + H₂ →

#ed_*"a#g#bk_9+#ed#bk:_*#g:^9`

Ion notation

The charge of an ion is recorded after the right superscript sign.

In the case of 2-, 3- and more positive ions, the * sign should be preceded by the separator !.

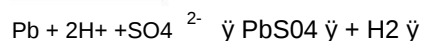
Fe + 2H⁺ + 2Cl⁻ → Fe²⁺ + 2Cl⁻ + H₂

→ #ed_*"a#g[*_*"a#bk[,_9+

9+#ed["a!*_*"a#bk[,_9`

Formation of insoluble salt
Molecular notation
Pb + H₂SO₄ → PbSO₄ ↓ + H₂ →

#Oa_.*^g:rn._9+^OaRn.^9_.*#g:^9`

Ionic record

#Oa_*"a#g[*_*^rn.["a,_9+
9+^OaRn.^9-_*#g:^9`

Reversible reaction K+ +

^JNG_ 9+#j[*_*^NG[,

Dissociation reaction

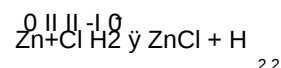
^gbnng_; 9+&^g:n-^gbnn[,_*#g[*

In the reaction record, the long arrow pointing to the right and left is used, written under the expression 77

; 9+&^g:n- - long arrow pointing left and right written under the expression.

Oxidation and reduction reactions

Oxidation states in red.—ox. reactions are written only in Roman numerals after the element symbol in round brackets.

Roman numeral

#ym1(?_*"a#g1#H?#bk1,#h?_ 9+
9+#ym1#HH?#bk:1,#h?_*#g:1(? `

The oxidation states in the examples below are given in shorthand notation.

Electron balance in oxidation-reduction reactions

#`k1(?_,"b&d9_9+#@k["b!*1#hhh?

Reaction sequence diagram

In chemistry problems, a sequence of reactions is often used for the student to decipher. The "+" sign indicates a compound added to the reaction, the "-" sign indicates a compound eliminated from the reaction.



#`_1*#bk:?_9+#a_1,#g#Bk?_9+#b

Unit notation [chem.]

78

In chemical notation, units are written in accordance with mathematical and physical notation. The unit designation is preceded by the name sign.

6z - unit sign, name sign.

g

mol

dm³

g/mol

6f 6lnk 6cl/9

6f7lnk

12g

5.6mol

3g/dm³

"a6f "d+e6lnk "b6f7cl/9

Solution concentration percentage

To calculate the percentage concentration of a solution, the following formula is used:

$$c_p = \frac{m_s}{m_r} \cdot 100\%$$

&b0o_<l0r7l0q_-"`ii"i(

When calculating the percentage concentration of a solution obtained by mixing solutions of different concentrations, the following formula is used:

$$c_{p1} = \frac{m_{s1}}{m_{r1}} \cdot 100\%$$

&b0o0+_<l0r0+7l0q0+_-"`ii"i(

or full record

&b0o0+_<:l0r0+_7 _l0q0+;- "`ii"i(

The example uses the signs used to write fractions⁷⁹:

:z - beginning of fraction (numerator) ;z - end

of fraction (denominator)

7 - fraction bar

and the sign of the right-hand indicator⁸⁰:

z0z

⁷⁸ The notation of units is detailed in the chapters of the Guide – *Units and operations on units and Verbal symbols, designation sign.*
Chapter 79 of the Guide – *Fractions.*

⁸⁰ Guide Chapter – *Key signs for exponents and right-hand indices.*

GENERAL PATTERNS

In organic chemistry, general formulas are used for groups of compounds, e.g. hydrocarbons, alcohols.

C_nH_{2n+2}

#b0&m4#g0"a&m!*"a

$C_nH_{2n+1}OH$

#b0&m4#g0"a&m!*" 4#n#g

The sign ! is a sign filling.

SEMI-STRUCTURAL AND STRUCTURAL PATTERNS**Bonds between atoms**

with

horizontal single

vertical single

+zzjz

from#from

horizontal double

vertical double

pattern

z^kz

triple horizontal

[zz

right oblique single

z0z

left oblique single

z[[z

right oblique double

z00z

left oblique double

Structural formulas of inorganic compounds

HCl	H - Cl	
		#g + #bk
WHAT	C = O	
		#b#j#N
N ₂	N ≡ N	
		#M^k#M
Ca(OH) ₂	HO-Ca-OH	
		#g#n + #b` + #n#g
or	H - O - Ca - O - H	
		#g + #n + #b` + #n + #g
Al ₂ O ₃	O = Al - O - Al = O	
		#n#j#`k + #n + #@k#j#n
SO ₃		
O = S = O		#N#j#R#j#N W
		#N

Structural and semi-structural formulas of organic compounds.

We write the formulas of hydrocarbons in summary and semi-structural terms.

In the case of long hydrocarbon chains, the notation can be shortened by placing the CH₂ group in round brackets and after the bracket in the subscript, a number specifying the number of these groups should be written. If it is an isomeric compound, the substituent should be placed in round brackets, without a bond sign at the carbon atom to which it is attached. The proposed Braille notation allows for the notation of two substituents at the same carbon atom or the notation of a substituent having an additional substituent.

C₂H₅COOH propanoic acid

CH₃ - CH₂ - COOH

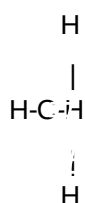
^bg9 + ^bg: + ^bnng #b#g9 or
 + #b#g: + #b#n#n#g

C₁₀H₂₂ n-decane

CH₃ - (CH₂)₈ - CH₃

^bg9 + 1^bg:?' + ^bg9

CH₄ methane:

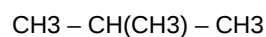


+#b +#g

2-methylpropane (isobutane):

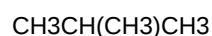


^bg9 +^bg +^bg9 |
j



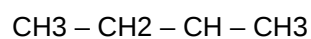
^bg9 +^bg1^bg9? +^bg9

or



#b#g9#b#g1#b#g9?#b#g9

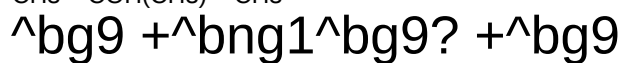
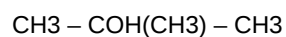
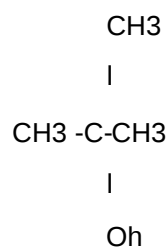
C₅H₁₂ 2-methylbutane:



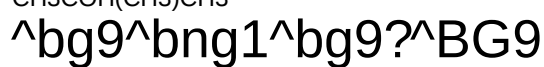
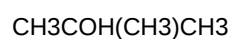
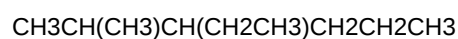
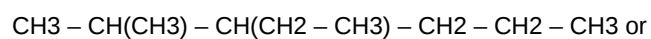
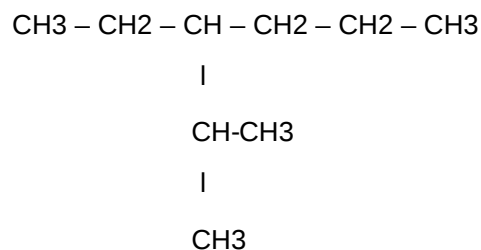
^bg9 +^bg: +^bg +^bg9

or

#b#g9 +#b#g: +#b#g +#b#g9

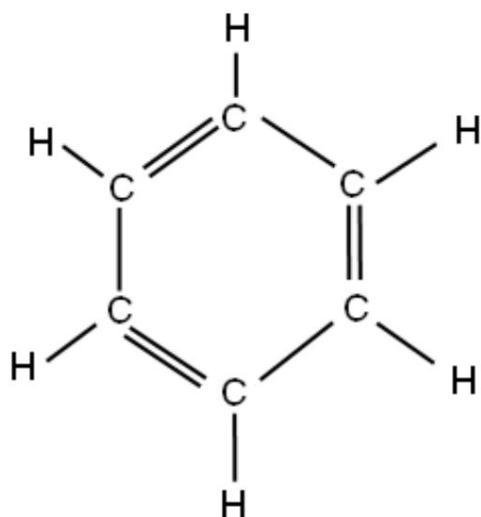
2-methyl-propan-2-ol:

or

**3-ethyl-2-methylhexane**

Ring compounds In the

structural formulas of ring compounds, horizontal and vertical bonds are used, as well as right- and left-oblique.

Benzene - structural formulas

#g

+#g

#g

+#g

Braille notation for ring compounds uses three levels of information: 1. The ring compound and the number of atoms constituting the ring.
2. Location and type of substituents.
3. Position of double bonds.

The following characters are used for this:

%0

a compound with a ring structure (single-ring or composed of several rings connected by bonds)

\$

polycyclic compound (rings are joined by common carbon atoms)

#0 in combination with a number indicates the number of atoms that make up the ring

4

a sign to close a piece of information

4

a complex sign of closing the whole pattern

%0#`

aromatic compound

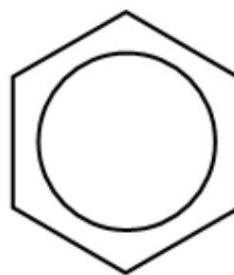
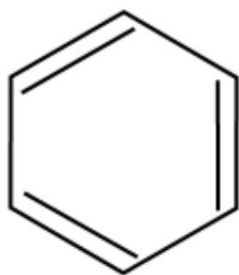
": "9

second level of information

third level of information

Benzene – one of the forms of Kekule's formula

simplified pattern



%#o"e4
"9_#j":bd*` 4

%#`o"e 4

Level 1: informs about the ring structure of the compound. %#o"e

means that it is a six-membered ring compound. %#`o"e 4 means that it is a six-membered aromatic compound.

Level 2: not used because there are no substituents.

Level 3: informs about the position of double bonds in the chain.

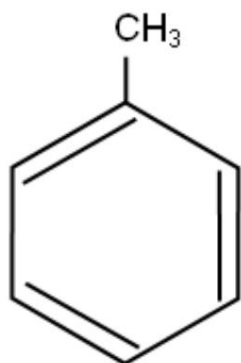
The record starts with the information level number "9", then after the blank character you should put the binding character double bond and then without a blank sign, after the number sign, write down the numbers of the carbon atoms between which the double bond is (the first digit reduced, the second without a reduction and the next ones according to the same principle)

e.g. #j":bd*` means that the double bonds are between carbon atoms 2 and 3, 4 and 5 respectively and 6 and 1.

Information levels 1 and 2 end with a closing character 4.

The whole pattern ends with a complex closing sign 4.

Toluene



%#o"e4
":_"+1^bg9?4 "9_#j"-
a9c>e 4

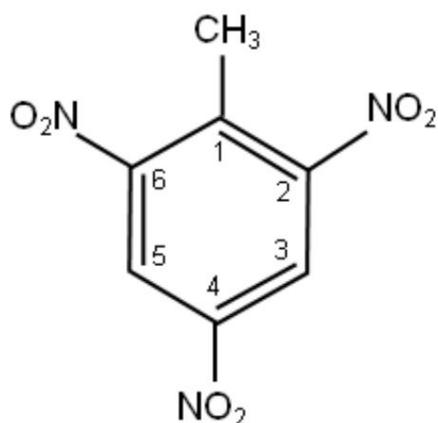
When writing the formulas of benzene derivatives, it is possible to use a simplified formula specifying the substituents and their locants (level 2 information).

Level 2 : informs about the location and type of substituents.

The record starts with the information level number ":", then after an empty character, record after the character numeric, reduced digit, the number of the carbon atom with the substituent and, without a blank sign, the formula of the substituent placed in round brackets.

For example "+1^bg9?4" means that a methyl group is attached to the first carbon atom. The subsequent substituent formulas are written analogously and without blank characters.

2, 4, 6 – trinitrotoluene (TNT, TNT)



%#o"e4

":_"+1^bg9?":1^mn:?.1^mn:?! !"*1^mn:?4
"9_#j"+a9c>e 4

In the case of a larger number of identical substituents, the notation can be shortened by giving each time after the number sign, in reduced notation, the carbon atom number with the substituent formula. The formula of a substituent enclosed in round brackets ends with a reduced number informing about the number of substituents.

%#o"e4

":_"+1^bg9?":."*1^mn:?94 "9_#j"+a9c>e
4

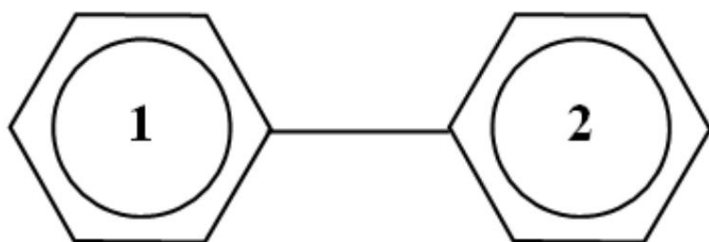
or

Simplified notation

%#`o"e4

":_"+1^bg9?":."*1^mn:?9 4

Diphenyl - two rings are connected by a bond and do not have a common side, so the formula in Braille it starts with the % sign.



%#`o"+e +#`o":e 4

Polycyclic Compounds In

Braille, polycyclic compounds are numbered by each ring. In each ring, the carbon atoms are numbered clockwise starting from the top carbon atom.

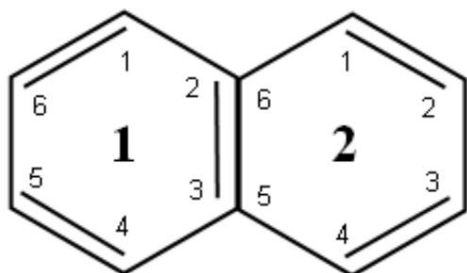
The record starts with the after the numeric sign, the number of the ring being described is given

character #o, in lower case, without a blank character. Then, between the characters U ", the number is given in lower case adjacent ring. Next, the numbers of the pairs of carbon atoms are given, listed clockwise, in the same notation as for bonds (the first resulting from the numbering in the described ring – in the lowercase notation, the second resulting from the numbering in the adjacent ring – in the uppercase notation).

For greater clarity, the description of subsequent rings starts on a new line.

If a substituent is attached to the ring, the ring number is given followed by the number of the carbon atom to which the substituent is attached.

Naphthalene – a two-ring compound – full notation.



```
#b____#b____#b
_j____w____j
#b____#b____#b
```

```
$"a#o":e&
____#o"+eU:":e9d4
"9_#o"+e#j":bd*`&
____#o":e#j"+a9c>e 4
```

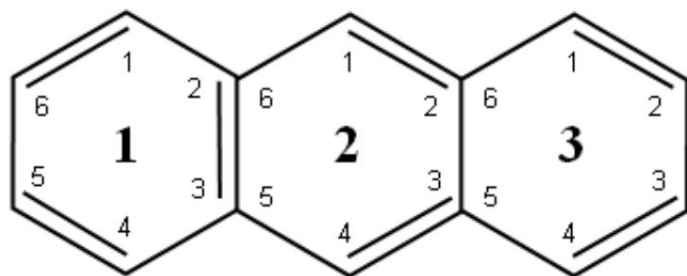
Naphthalene – short form.

To specify a common side, only the number of the carbon atom starting the common side is given.

Then the first common pair of carbon atoms is given in the same notation as before (top, bottom). When describing rings, the number of ring members is omitted.

```
$"a#o":e&
____#o"+U:":e4
"9_#o"+#j":bd*`&
____#o":#j"+a9c>e 4
```

Anthracene – a tricyclic compound – full entry.



\$"b#o"9e&

___#o"+U:":e9d&

___#o":U+ ">b*aU9":e9d&

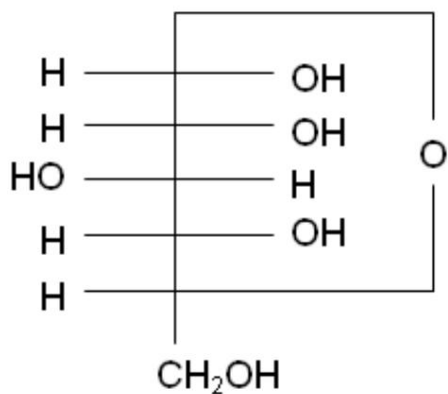
___#o"9U: ">b*a4

"9_#o"+#j":bd*`&

___#o":#j"+a9c>e&

___#o"9#j"+a9c 4

Glucose: β -D-glucopyranose – ring form – Fischer notation.



To close the ring of a monosaccharide molecule in Braille, a row of dots is used.

+ + + + + + +

_#G + #B

_#G + #B

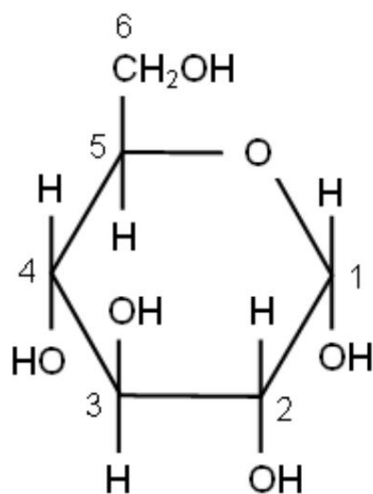
^GN + #B

_#G + #B

_#G + #B + + + + + ++

____^BG:NG

Glucose: β -D-glucopyranose – ring form – Haworth notation.



_____ ^bg:ng

+ + + + #n

+ + + + #b

%#o"e4

":_ "+10^ng?1{#g?":10^ng?1[#g?!

___!"910#g?1{^ng?".10^ng?1{#g?!

___!">10#g?1{"^bg:ng? + #n 4

Fructose - β -D-fructofuranose - carbon atoms are numbered counterclockwise (to the left). In Braille, this is indicated by the character 9, understood as an auxiliary arrow.

__#g_0#g_____^gn_[^bg:ng_____

+ + +

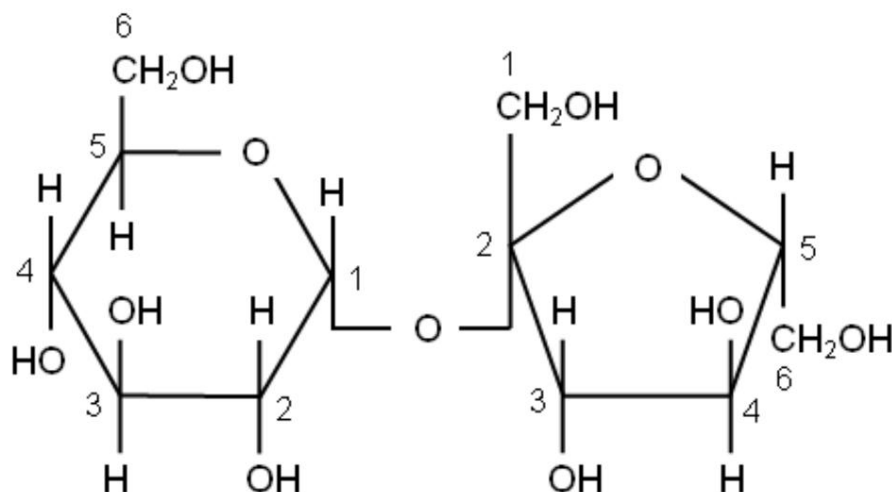
%#o"d4

":_ 9":10#g?1["+^bg:ng?!

__!"910^ng?1[#g?".10#g?1[^ng?!

__!"d10"*^bg:ng?1[#g? +#n 4

Sucrose - in the fructose ring the carbon atoms are numbered counterclockwise (to the left). This is indicated by the sign 9 understood as an auxiliary arrow.



%#o"+e +#n +#o":d4

":_#o"+e"+10#n#o":d?1{#g?!

____!":10^ng?1[#g?"910#g?!

____!1[^ng?".10^ng?1[#g?! ____!">10#g?

1["*^bg:ng? +#n& ____#o":d 9":10#n#o"+e?1["+!

____!^bg:ng?"910^ng?1[#g?!

____!"10#g?1[^ng?">10!

____!"*^bg:ng?1[#g? +#n

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