

**THE NEMETH BRAILLE CODE
FOR MATHEMATICS
AND SCIENCE NOTATION
1972 REVISION**

Compiled Under the Authority of the
**AMERICAN ASSOCIATION OF WORKERS FOR THE BLIND
ASSOCIATION FOR EDUCATION OF THE VISUALLY HANDICAPPED
and the
NATIONAL BRAILLE ASSOCIATION**

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AAWB-AEVH-NBA ADVISORY COUNCIL TO THE BRAILLE AUTHORITY

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FOREWORD TO 1972 REVISION

THE NEMETH CODE OF BRAILLE MATHEMATICS AND SCIENTIFIC NOTATION, 1965 initiated sound principles and procedures for the presentation of braille equivalents for the complex signs and configurations of ink-print mathematical and scientific notation. The effectiveness of the Code has been amply demonstrated through its application by transcribers in producing a wealth of technical material to meet the requirements of students at all levels of educational pursuits.

At the time of publication, it was apparent that the Code would require further updating and refinement in order to assure the faithful transference from ink print to braille as new modes of scientific notation were introduced. As was anticipated, problems in interpretation and clarity were encountered when the Code was put into actual use. The comments, criticisms and suggestions from students, teachers and transcribers were taken under consideration in the revision of the Code.

Under the able tutelage of Dr. Abraham Nemeth, the members of the AAWB-AEVH Braille Authority and its Advisory Committee on Mathematical and Scientific Notation entered upon a joint effort in bringing forth a Revised Code which could withstand the test of use and time. As work progressed, however, it became increasingly evident that, because of the complexity of the subject matter and because of the many techniques employed by authors and publishers, substantial research would be required in expanding the Code to its fullest effectiveness. In recognition of this fact, the national Advisory Council to the Braille Authority applied for a planning grant from Social and Rehabilitation Services of the Department of Health, Education, and Welfare. The American Printing House for the Blind was designated as the recipient of the grant, known as the "Braille Codes Pilot Project", which is geared to bring into focus the need for fuller research in all braille codes. Upon the basis of this study, application for a research grant will be made and, if approved, all braille codes will be considered in detail in the endeavor to bring them to maximum completeness and efficiency.

The 1972 revision of THE NEMETH BRAILLE CODE FOR MATHEMATICS AND SCIENCE NOTATION provides students and transcribers with a well-drawn, logical system of braille notation which insures a faithful presentation of signs and usages employed in technical texts. The changes which have been incorporated will convey to the reader a realistic picture of the ink-print text and will equip the transcriber with the necessary signs and rules of procedure for a more exact braille transcription.

Grateful acknowledgement is accorded the following persons for their major contribution in the joint effort in developing and refining the revised Code.

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THE NEMETH BRAILLE CODE FOR MATHEMATICS AND SCIENCE NOTATION 1972 REVISION

ORIENTATION

§1. Description:

a. This Braille Code for Mathematics and Science Notation has been prepared to provide a system of symbols which will allow technical literature to be presented and read in braille. The Code is intended to convey as accurate an impression as is possible to the braille reader of the corresponding printed text, and this is one of its principal features. When the braille reader has a clear conception of the corresponding printed text, the area of communication between himself and his teacher, his colleagues, his associates, and the world at large is greatly broadened. *A test of the accuracy with which the Code conveys information from the print to the braille text is to effect a transcription in the reverse direction. The amount of agreement between the original printed text and one transcribed from the braille is a measure of the Code's accuracy.*

b. A careful distinction is maintained between the meaning which a printed sign has and the sign itself. Sometimes the name of a sign derives from the mathematical meaning which it has. Simple examples are the *plus* sign, the *decimal point*, and the *percent* sign. Other signs have names which are descriptive of the signs themselves, such as *arrow*, *vertical bar*, and *diagonal line*. Still others have names in accordance with the way in which they are read aloud, such as *is less than*, *is contained in*, or *is an element of*. Some signs have no name at all. Of course, the majority of signs, particularly at the lower levels of mathematics, are signs about which there is universal agreement as to their meaning, and these constitute the core about which has grown the modern system of mathematical notation. However, at the more advanced levels, technical writers have, with increasing frequency, been assigning new and unusual meanings to many of the signs which have long been "standard."

c. Although the Code is intended to be as complete as possible, finality can never be achieved by any code. In the course of the rapid development in the fields of science and technology, new signs are constantly being devised and old ones modified. At appropriate places, rules and suggestions are presented for transcribing signs for which no specific provision exists at present.

§2. Organization:

a. In presenting this Code, the needs of both the transcriber and of the reader have been considered. While the rules of the Code have been formulated primarily for the benefit of the braille reader, they are nevertheless presented in a manner designed to ease the transcriber's task of following these rules. The problem of transcription is intrinsically more difficult than the problem of reading; the transcriber must actually recall to mind the specific symbols which must be used and the rules which govern their use, whereas the braille reader must only recognize the symbols which he encounters and be only slightly aware of the underlying rules.

b. This presentation is organized into rules. Where appropriate, each rule begins with a list of signs and their corresponding symbols for quick reference. The body of the presentation is organized into sections which are consecutively numbered and captioned. The sections contain rules, explanations, and examples of the use of the Code. It is intended that the examples be sufficiently definitive so that they may be imitated with confidence in parallel situations. The parenthetical descriptions below the examples are intended to supplement the actual signs shown in the ink print copy. The examples in this presentation are drawn principally from the central core of pure mathematics. Other scientists will find few examples from their fields. Nevertheless, the symbols, rules, and constructions of the basic Code apply with equal force to those fields. Following the rules, there is an INDEX OF BRAILLE SYMBOLS the entries of which have been categorized in accordance with the standard arrangement of the sixty-three braille symbols.

c. Throughout this presentation, the word *sign* is consistently used in referring to a character or sequence of characters in ink print, whereas the word *symbol* is used in referring to a character or sequence of characters in braille.

d. In this text, mathematical or literary material which often appears in italics or other type in ink-print textbooks has been printed in regular type. Italic and other type forms have been used only where such type is required to illustrate a rule.

e. Although 41 cells may be used in transcribing technical works, examples in this text have been shortened to conform to the space available on the ink-print lines.

§3. Interpretation: *It is important that this presentation be accepted quite literally and that no meaning be imputed to the rules and principles which is not expressly stated or directly implied. It may sometimes appear quite arbitrary that a particular sign has been classified in a section which the reader's past experience or training indicates is inappropriate. For the purposes of this Code, however, the transcriber or teacher must accept the classification as well as the rules herewith presented, past experience or technical training notwithstanding.*

In certain situations it may be felt that some constructions are excessively long and there may be a temptation to shorten the construction by the use of a symbol of one's own invention. However, *the transcriber is enjoined against yielding to this temptation.* The Code has been formulated in such a way that the same construction gives the same information to the braille reader from elementary through the most advanced mathematics. Therefore, tampering with the constructions presented herein would have the effect of destroying this uniformity. Signs which for many decades have been exclusively associated with college and graduate mathematics have in recent years been filtering down to high school and grade school levels. Thus, the set operations such as *union*, *intersection*, and *inclusion*, which were traditionally encountered for the first time by a mathematics major in his junior year in college, have now become fairly commonplace at the fourth or fifth grade level, and are first met even earlier. In addition, grade school and high school mathematics are now being presented with considerably more rigor than heretofore, and shades in meaning are being preserved and even emphasized by the use of distinct signs having similar, but not equivalent, meanings. In keeping with this spirit, the Code furnishes distinct braille symbols corresponding to distinct signs in ink print. In particular, at the lower levels of mathematics, this Code maintains a distinction between the horizontal and diagonal fraction lines, and between the *dot* and the *cross* which signify multiplication. Signs which have separate identities in ink print should be represented by distinct symbols in braille.

§4. Technical and Non-Technical Texts:

a. The designation *non-technical* implies only the absence of mathematical or scientific notation; a work in law or medicine may be quite technical in those fields, but must be regarded as non-technical in the sense just mentioned.

b. *Partially technical* works include science books written for the layman or textbooks in other fields which use mathematical terminology and notation. Such works are characterized by the use of an occasional mathematical sign or a small number of such signs. In works of this kind, the mathematical signs may be treated as in English Braille. This procedure is particularly suitable when there is no intention that the reader should manipulate such signs for the purpose of solving equations or performing computations. Sometimes, however, the replacement of a sign by a corresponding word is not practical, especially when an aggregate of such signs appears in an arrangement which is unusual from the literary point of view. In such cases, the transcriber should use the symbols and the rules of this Code with a note to the braille reader that this is being done. A list of the mathematical symbols being used should be included at the beginning of only the braille volume in which they occur.

c. *Technical* works are those in the fields of mathematics, statistics, physics, or chemistry. In such works the symbols and rules of this Code must be used. They must also be used in works in other fields which make strong use of mathematical signs and modes of expression. In all technical works the transcriber must indicate at the beginning of each volume by means of a transcriber's note that the work has been transcribed in Nemeth Code, giving the year the code was adopted. Even when the Nemeth Code is used, title pages must be transcribed as in English Braille without the use of Nemeth Code symbols, except for items which contain mathematical expressions for which it would be inappropriate or impractical to use English Braille.

d. It is recommended that machines be set for a braille line of 41 cells when transcribing technical works.

RULES

RULE I—BRAILLE INDICATORS

Alphabetic Indicators

English-Letter	⠠
German-Letter	⠤
Greek-Letter	⠢
For standard letters	⠠
For alternative letters	⠠⠠
Hebrew-Letter	⠠⠠
Russian-Letter	⠠⠠

Arrow Direction Indicators

Depresses Nearer Arrowhead by 45 Degrees	⠠
Elevates Nearer Arrowhead by 45 Degrees	⠤
Makes Nearer Arrowhead Point Up	⠠
Makes Nearer Arrowhead Point Down	⠤

Arrow Types: Boldface

⠠

Cancellation Indicators

Opening	⠠
Closing	⠤

Capitalization Indicators

Single	⠠
Double	⠠⠠

Carried Number Indicator for Addition
(varying in length)

⠠⠠⠠

Fraction Indicators**Simple**

Opening



Closing

**Complex**

Opening



Closing

**Hypercomplex**

Opening



Closing

**Fractional Part of a Mixed Number**

Opening



Closing

**General Reference Indicator****Level Indicators**

Base Line



Superscript



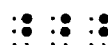
Superscript with Superscript



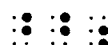
Superscript with Subscript



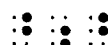
Superscript with Superscript with Superscript



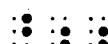
Superscript with Superscript with Subscript



Superscript with Subscript with Superscript



Superscript with Subscript with Subscript



Subscript



Subscript with Superscript



Subscript with Subscript	⠠⠠⠨
Subscript with Superscript with Superscript	⠠⠠⠨⠠⠠⠨
Subscript with Superscript with Subscript	⠠⠠⠨⠠⠠⠨
Subscript with Subscript with Superscript	⠠⠠⠨⠠⠠⠨
Subscript with Subscript with Subscript	⠠⠠⠨⠠⠠⠨
Modification Indicators	
Multipurpose	⠠⠠
Directly Over	
First order	⠠⠠
Second order	⠠⠠⠠
Directly Under	
First order	⠠⠠
Second order	⠠⠠⠠
Superposition	⠠⠠
Termination	⠠⠠
Multipurpose Indicator	⠠⠠
Numeric Indicator	⠠⠠
Punctuation Indicator	⠠⠠
Radical Indicators	
Index-of-Radical	⠠⠠
Order-of-Radical	
First inner radical	⠠⠠
Second inner radical	⠠⠠⠠
Third inner radical	⠠⠠⠠⠠
Termination	⠠⠠

Shape Indicators

Shape	⠠
Structural Shape-Modification	⠠
Interior Shape-Modification	⠠
Filled-In Shape	⠠
Shaded Shape	⠠
Termination	⠠

Termination Indicator

⠠

Type-Form Indicators for Letters, Numerals, and Compound Expressions

Boldface Type	⠠
Italic Type	⠠
Sanserif Type	⠠
Script Type	⠠

Type-Form Indicators for Words, Phrases, and Mathematical Statements

Opening Boldface Type	⠠
Opening Italic Type	⠠
Closing Boldface Type	⠠
Closing Italic Type	⠠

§5. **Concept of Braille Indicators:** Mathematical expressions are represented in ink print by the use of arbitrary signs among which are the digits, the lower-case and capitalized letters of several alphabets, the script, italic, and boldface forms of these same letters, as well as numerous signs of operation, signs of comparison, signs of grouping, and many other signs serving the miscellaneous requirements of mathematical and scientific expression. Furthermore, mathematical significance is imparted not only by these signs separately, but by their collective arrangement on levels above or below a reference line of writing, as well as by their disposition above or below a fraction line. With only sixty-three distinct braille characters available, sixty-four if the space is counted, the accomplishment of this Code is to make provision for the representation of all these signs, as well as to give an indication of their arrangement.

It is, of course, impossible to establish a one-to-one correspondence between the sixty-three braille characters and the hundreds of signs used in modern mathematics. It is also impractical, as a general procedure, to imitate the arrangement of these signs at various levels relative to a reference line of writing or to a fraction line. Accordingly, the Code presented in the following pages is characterized by the use of a system of braille indicators. The braille indicators in this Code play the same role as do the composition signs

of English Braille. In both systems, the braille indicators or the signs of composition correspond to no sign in ink print; however, they have the power of imparting meaning to the braille symbols with which they are associated. While there are only a few signs of composition in English Braille, there are many in this Code. By their use it is possible to represent the numerous type forms and alphabets used in ink print and to convey the "two-dimensional" information contained in ink print through the medium of the braille system whose nature is essentially "one-dimensional."

§6. Spacing with Braille Indicators: No space should be left between a braille indicator and the symbol or expression to which it applies. In addition, the punctuation indicator, level indicators, and modification indicators apply both to the material which precedes as well as to the material which follows them. There are special spacing rules for the type-form indicators for words, phrases, and mathematical statements (see §33). Examples illustrating this spacing rule are found throughout the Code.

RULE II—NUMERIC SIGNS AND SYMBOLS

Numeric Indicator

⠠

Arabic Digits (Nemeth Code)

0	1	2	3	4	5	6	7	8	9
⠼	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠

Comma (mathematical)

American , ⠸

Continental . ⠸

Decimal Point

American . ⠸

Continental , ⠸

§7. Representation of Arabic Numerals:

a. Digits are represented in two ways: as in English Braille, and as in the Nemeth Code. The digits in English Braille are represented by the letters "j" and "a" through "i". The digits in the Nemeth Code are represented by the symbols whose configurations correspond to these same letters, but which occupy the lower portion of the braille cell.

b. Even when a work is transcribed in the Nemeth Code, when at all practical or appropriate, all numerals on title pages must be transcribed as in English Braille. Numerals at the corners of pages and at the ends of page-separation lines must also be transcribed as in English Braille. English Braille numerals must be used when the technique of "keying" (§187) is employed. In all other cases, including contents pages, forewords, introductions, page references, footnotes, indices, and bibliographies, the numerals of the Nemeth Code must be used.

§8. Comma, Decimal Point:

a. The transcriber should be alert to the possibility that variant forms of the comma and decimal point are sometimes employed, particularly in books published outside of the United States. Although the ink print signs for the comma and decimal point differ from

those used in the United States, this difference is not reflected in the braille transcription. However, a transcriber's note should be included at the beginning of the book to inform the reader of the continental usage in the ink-print edition.

- (1) 1,378 ⠠⠠⠠⠠⠠⠠⠠
(American usage of comma)

- (2) 1.378 ⠠⠠⠠⠠⠠⠠⠠
(Continental usage of comma)

- (3) 3.76 ⠠⠠⠠⠠⠠⠠
(American usage of decimal)

- (4) 3,76 ⠠⠠⠠⠠⠠⠠
(Continental usage of decimal)

b. *The comma*, American or Continental, which is interior to a numeral, and which is used to partition the numeral into short regular segments, must be regarded as a numeric symbol. As such, the comma is subject to the rules for transcribing numerals.

- (1) 1,478 ⠠⠠⠠⠠⠠⠠
(the comma is a numeric symbol, not a punctuation mark)

- (2) 100, 200, 300 ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
(the commas are punctuation marks, not numeric symbols)

c. *The decimal point*, American or Continental, should be regarded as a numeric symbol only when it is associated with a numeral. An omission symbol must not be regarded as a numeric symbol. As a numeric symbol, the decimal point is subject to the rules for transcribing numerals.

- (1) .35 ⠠⠠⠠⠠
(the decimal point is a numeric symbol)

- (2) 3.14 ⠠⠠⠠⠠⠠⠠
(the decimal point is a numeric symbol)

- (3) .2a₁a₂a₃ ⠠⠠⠠⠠⠠⠠⠠⠠⠠
(the decimal point is a numeric symbol)

- (4) .a₁a₂a₃ ⠠⠠⠠⠠⠠⠠
(the decimal point is not a numeric symbol)

- (5) .1 + .2 = — ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
(the first two decimal points are numeric symbols)

(12)
$$\frac{\frac{1+3}{4+5}}{\frac{3+4}{5+6}}$$

(a complex fraction)

(13)
$$\frac{(1-x) \frac{d}{dx} (2x) - 2x \frac{d}{dx} (1-x)}{(1-x)^2}$$

(a hypercomplex fraction)

(14) -1

(minus 1)

(15) $-.3$

(minus three tenths)

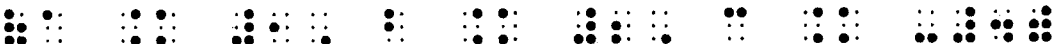
- (4) See page 15¹. (1 follows the general reference indicator; the 1 is raised in ink print)
- (5) †3 (dagger 3)

e. The numeric indicator must be used after any of the type-form indicators, or after making a transition from non-regular to regular type within the same numeral. It must also be used after the interior shape-modification indicator.

- (1) *3* (italicized 3)
- (2) **0** (boldface zero)
- (3) *.3* (italicized three tenths)
- (4) *2* (script 2)
- (5) **4356** (the first two digits in boldface type, the last two in regular type)
- (6) ⑤ (5 enclosed within a circle)
- (7) ⑤ (5 enclosed within a square)

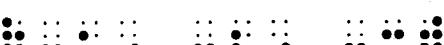
f. The numeric indicator must be used after a hyphen when the hyphen follows a word, an abbreviation, or a mark of punctuation. However, also see §11d.

- (1) 1-to-1 correspondence (1 follows the general reference indicator; the 1 is raised in ink print)
- (2) hydrogen-3 (3 follows the general reference indicator; the 3 is raised in ink print)
- (3) DC-7 (7 follows the general reference indicator; the 7 is raised in ink print)
- (4) B-49 bomber (4 follows the general reference indicator; the 4 is raised in ink print)
- (5) U-238 (2 follows the general reference indicator; the 2 is raised in ink print)
- (6) (2877-212 B.C.) (2 follows the general reference indicator; the 2 is raised in ink print)

- (8) $[\angle 1^\circ, \sin 1^\circ]$ 
(an "enclosed list"; meets all requirements)
- (9) $(a, b, \dots)$ 
(an "enclosed list"; meets all requirements)
- (10) $(x + 1, x + 2, ?, ?, x + 5)$ 
(an "enclosed list"; meets all requirements)
- (11) $(x = 1, 2, \dots, 10)$ 
(not an "enclosed list" according to v)
- (12) $(a = 1, b = 2, c = -4)$

(not an "enclosed list" according to v)
- (13) $(u, v; x, y)$ 
(not an "enclosed list" according to vi)
- (14) $(1\ 2\ 3)$ 
(not an "enclosed list" according to vi)

§11. **Non-Use of the Numeric Indicator:** It must not be assumed that because a symbol is numeric that the numeric indicator must be used with that symbol. The numeric indicator must not be used preceding a numeric symbol under the following circumstances:

a. The numeric indicator must not be used at the *beginning* of an item which is part of an "enclosed list" as defined in §10 above, even if such an item has been run over to another line. However, if any item in an "enclosed list" is a numeral in a type form other than regular type, that item requires the numeric indicator.

- (1) $[0, 1]$ 
- (2) $(-1, -2, -3)$ 
- (3) $(1 + h, 2 + k, 0)$ 
- (4) $(0, -1, \pm 2)$ 
- (5) $(2 \sin 30^\circ, 3 \cos 60^\circ)$

(the numeric indicator is required before the 30 and the 60 because these are *not the beginning* of their respective items)

§12. Long Numerals: Long numerals that cannot be completely accommodated on one braille line may be divided and run over to another line. Such a division must be made after a comma, if present, and a hyphen must be supplied. The numeric indicator must be used as the first braille symbol of the braille line to which the numeral has been run over.

- (1) 100,000,000,000,-
000
- (2) 1000000000000-
000

§13. Representation of Numerals to Non-Decimal Bases:

a. When a system of numeration is to a base other than 10, a common technique for providing additional digits is to use letters, either lower-case or capitalized, in addition to the ten Arabic digits. When this technique is used, the transcriber must use only lower-case letters. If capitalized letters are used in ink print, the transcriber must indicate this fact in a transcriber's note.

- (1) 13TE7
- (a base-12 numeral in which T represents 10 and E represents 11)
- (2) 3FFE2
- (a base-16 numeral in which E represents 14 and F represents 15)

b. Another common technique for providing additional digits is to use standard or arbitrary signs to supplement the ten Arabic digits. Authors sometimes give names to these signs. For example, X (dek) represents 10 and E (el) represents 11. In this case, the transcriber must devise one-cell symbols for these signs, preferably chosen from among the letters of the English alphabet, and must insert a transcriber's note to specify the meanings which have been assigned to these symbols. The transcriber's note must include a drawing of any sign for which there is no equivalent symbol in the Code.

- (1) 13XE7
- (a base-12 numeral in which X represents 10 and E represents 11. Here the transcriber has assigned X and E to 10 and 11)

c. Another common technique is to use an arbitrary set of signs which do not include Arabic digits. In this case, the transcriber must proceed as in b above.

- (1) @%\$
- (a base-3 numeral of three digits; here the transcriber has assigned @ to 0, % to 1, and \$ to 2)

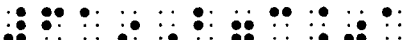
d. The one-cell symbols which the transcriber uses to represent the digits of a non-decimal numeration system must be regarded as numeric symbols. As such, these numeric symbols are subject to the rules for transcribing numerals.

(1) t2e4 

(a base-12 numeral; t and e are lower case in ink print)

(2) 3t.t8 

(a base-12 numeral containing a decimal point between the second and third digits; t is lower case in ink print)

(3) FA9,B7C.0A 

(a base-16 numeral containing a comma and a decimal point; F, A, B and C are capitalized in ink print)

§14. Ordinal Endings: (See §55d.)

§15. Plural and Possessive Endings: (See §39.)

§16. Numerals in Diagrams: In diagrams which contain numeric labels, the numeric indicator must be used. The space to accommodate the numeric indicator may often be gained by a sufficient enlargement of the diagram.

§17. Numerals in Table Entries: In tables whose entries consist entirely of numerals, the numeric indicator must be omitted. However, in tables whose entries are a mixture of words, numerals, letters, or other mathematical signs, the numeric indicator must be used. This rule applies only to the body of a table and not to the headings. Determinants and matrices are not to be regarded as tables. The minus symbol is not numeric so that, if it occurs in a table, the numeric indicator must be used throughout the table.

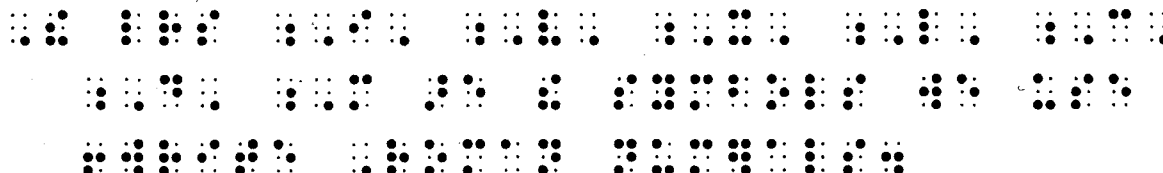
§18. Roman Numerals:

a. Capitalized Roman numerals must be transcribed using the single capital sign before one letter and the double capital sign before more than one letter. For the use of the English-letter indicator with Roman numerals, see §28c.

(1) I, II, III, IV, V.

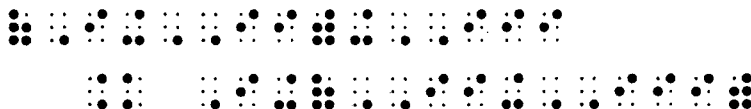


(2) The letters I, V, X, L, C, D, M are the symbols we use to write Roman numerals.



(3) VII + V = XII 

(4) (I + II) + III = I + (II + III)

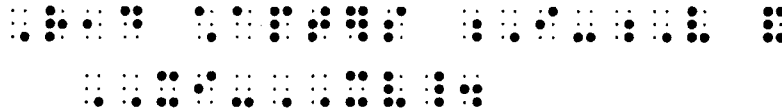


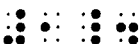
- (5) II' 
(prime sign follows the Roman numeral)

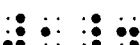
- (6) $\overline{\text{M}}$ 
(bar over M)

- (7) $V = 5, L = 50.$ 

- (8) Read chapters I-V and XI-XV.



- (9) 1. 
(I) 
(II) 

- (10) 1. 
I. 
II. 

b. When a Roman numeral consists of one or more lower-case letters it must be treated as though it were a "single letter" and, as such, the English-letter indicator must be used or not used in accordance with the rules governing the English-letter indicator (see §§26-28).

- (1) i, ii, iii, iv, v.



- (2) See pages v and vi.



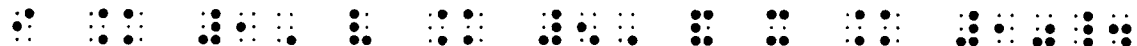
- (3) ¶a, §i and §ii.



- (4) vi + iv = x



- (5) i = 1, v = 5, and x = 10.



§20. Use of the Capitalization Indicator:

a. The capitalization indicator must be used to indicate the capitalization of a letter from any of the alphabets listed in Rule IV, except the Hebrew alphabet whose letters do not possess a capitalized form. This indicator must precede the letter concerned.

(1) **A** ⠠⠠⠠⠠⠠
(German capitalized ah)

(2) **Γ** ⠠⠠⠠⠠⠠
(Greek capitalized gamma)

b. For capitalized Roman numerals, see §18a.

c. For the capitalization of abbreviations, see §50.

§21. Non-Use of the Capitalization Indicator: Capitalization must not be used with a letter just because it begins a sentence, if the corresponding letter in ink print is uncapitalized.

(1) x is a number between 2 and 3.

⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

§22. Effectiveness of the Capitalization Indicator:

a. The effectiveness of the single capitalization indicator extends only to the letter which follows it, so that if each letter in a sequence requires capitalization, the capitalization indicator must be used with each of these letters individually.

(1) **△ ABC** ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
(triangle ABC)

b. The effectiveness of the double capitalization indicator in Roman numerals and in abbreviations extends to all of the letters which immediately follow it. However, a symbol other than a letter terminates its effect.

(1) **LL.D.** ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

(2) **III + V** ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

RULE IV—ALPHABETS**Alphabetic Indicators**

English (Roman) Letter ⠠⠠⠠⠠⠠

German-Letter ⠠⠠⠠⠠⠠

Greek-Letter

For Standard Letters



For Alternative Forms of Letters



Hebrew-Letter



Russian (Cyrillic) Letter



(For combinations of capitalization, alphabetic, and type-form indicators, see Appendix, page 208)

Alphabets

English (Roman) Alphabet

Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Sanserif capitalized	Braille equivalent
a	A	<i>a</i>	<i>A</i>	A	⠁
b	B	<i>b</i>	<i>B</i>	B	⠃
c	C	<i>c</i>	<i>C</i>	C	⠉
d	D	<i>d</i>	<i>D</i>	D	⠙
e	E	<i>e</i>	<i>E</i>	E	⠑
f	F	<i>f</i>	<i>F</i>	F	⠋
g	G	<i>g</i>	<i>G</i>	G	⠎
h	H	<i>h</i>	<i>H</i>	H	⠓
i	I	<i>i</i>	<i>I</i>	I	⠏
j	J	<i>j</i>	<i>J</i>	J	⠗
k	K	<i>k</i>	<i>K</i>	K	⠅
l	L	<i>l</i>	<i>L</i>	L	⠇
m	M	<i>m</i>	<i>M</i>	M	⠍

Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Sanserif capitalized	Braille equivalent
n	N	<i>n</i>	<i>N</i>	N	⠝
o	O	<i>o</i>	<i>O</i>	O	⠕
p	P	<i>p</i>	<i>P</i>	P	⠏
q	Q	<i>q</i>	<i>Q</i>	Q	⠒
r	R	<i>r</i>	<i>R</i>	R	⠗
s	S	<i>s</i>	<i>S</i>	S	⠎
t	T	<i>t</i>	<i>T</i>	T	⠞
u	U	<i>u</i>	<i>U</i>	U	⠥
v	V	<i>v</i>	<i>V</i>	V	⠺
w	W	<i>w</i>	<i>W</i>	W	⠽
x	X	<i>x</i>	<i>X</i>	X	⠭
y	Y	<i>y</i>	<i>Y</i>	Y	⠽
z	Z	<i>z</i>	<i>Z</i>	Z	⠵

German Alphabet

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
ah	a	A	<i>a</i>	<i>A</i>	⠁
beh	b	B	<i>b</i>	<i>B</i>	⠃
tseh	c	C	<i>c</i>	<i>C</i>	⠉
deh	d	D	<i>d</i>	<i>D</i>	⠙
eh	e	E	<i>e</i>	<i>E</i>	⠑
eff	f	F	<i>f</i>	<i>F</i>	⠋
gheh	g	G	<i>g</i>	<i>G</i>	⠗
hah	h	H	<i>h</i>	<i>H</i> or <i>L</i>	⠈
ee	i	I	<i>i</i>	<i>I</i>	⠊
yaht	j	J	<i>j</i>	<i>J</i> or <i>Y</i>	⠞
kah	k	K	<i>k</i>	<i>K</i>	⠅
ell	l	L	<i>l</i>	<i>L</i>	⠙
em	m	M	<i>m</i>	<i>M</i>	⠓

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
en	n	N	<i>n</i>	<i>N</i>	⠝
oh	o	O	<i>o</i>	<i>O</i>	⠕
peh	p	P	<i>p</i>	<i>P</i>	⠏
koo	q	Q	<i>q</i>	<i>Q</i>	⠒
err	r	R	<i>r</i>	<i>R</i>	⠗
ess	s	S	<i>s</i> or <i>ö</i>	<i>S</i>	⠎
teh	t	T	<i>t</i>	<i>T</i>	⠞
oo	u	U	<i>u</i>	<i>U</i>	⠥
fao	v	V	<i>v</i>	<i>V</i>	⠧
veh	w	W	<i>w</i>	<i>W</i>	⠪
iks	x	X	<i>x</i>	<i>X</i>	⠭
ypsilon	y	Y	<i>y</i>	<i>Y</i>	⠽
tset	z	Z	<i>z</i>	<i>Z</i>	⠵

Greek Alphabet (Standard)

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
alpha	α	A	<i>α</i>	<i>A</i>	⠁
beta	β	B	<i>β</i>	<i>B</i>	⠃
gamma	γ	Γ	<i>γ</i>	<i>Γ</i>	⠉
delta	δ	Δ	<i>δ</i>	<i>Δ</i>	⠙
epsilon	ε	E	<i>ε</i>	<i>E</i>	⠑

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
zeta	ζ	Z	<i>ζ</i>	<i>Z</i>	⠵
eta	η	H	<i>η</i>	<i>H</i>	⠈
theta	θ	Θ	<i>θ</i>	<i>Θ</i>	⠉
iota	ι	I	<i>ι</i>	<i>I</i>	⠊
kappa	κ	K	<i>κ</i>	<i>K</i>	⠅

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
lambda	λ	Λ	λ	Λ	⠠⠠⠠
mu	μ	Μ	μ	Μ	⠠⠠⠠
nu	ν	Ν	ν	Ν	⠠⠠⠠
xi	ξ	Ξ	ξ	Ξ	⠠⠠⠠
omicron	ο	Ο	ο	Ο	⠠⠠⠠
pi	π	Π	π	Π	⠠⠠⠠
rho	ρ	Ρ	ρ	Ρ	⠠⠠⠠

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
sigma	σ	Σ	σ	Σ	⠠⠠⠠
tau	τ	Τ	τ	Τ	⠠⠠⠠
upsilon	υ	Υ	υ	Υ	⠠⠠⠠
phi	φ	Φ	φ	Φ	⠠⠠⠠
chi	χ	Χ	χ	Χ	⠠⠠⠠
psi	ψ	Ψ	ψ	Ψ	⠠⠠⠠
omega	ω	Ω	ω	Ω	⠠⠠⠠

Hebrew Alphabet

Hebrew letters do not possess a capitalized form.

Name of letter	Ordinary	Script	Braille equivalent	Name of letter	Ordinary	Script	Braille equivalent	Name of letter	Ordinary	Script	Braille equivalent
aleph	א	א	⠠⠠⠠	teth	ט	ט	⠠⠠⠠	feh	פ	פ	⠠⠠⠠
veth	ב	ב	⠠⠠⠠	yod	י	י	⠠⠠⠠	tsadi	צ	צ	⠠⠠⠠
gimel	ג	ג	⠠⠠⠠	chaph	כ	כ	⠠⠠⠠	koph	ק	ק	⠠⠠⠠
daleth	ד	ד	⠠⠠⠠	lamed	ל	ל	⠠⠠⠠	resh	ר	ר	⠠⠠⠠
heh	ה	ה	⠠⠠⠠	mem	מ	מ	⠠⠠⠠	sin	ש	ש	⠠⠠⠠
vav	ו	ו	⠠⠠⠠	nun	נ	נ	⠠⠠⠠	thav	ת	ת	⠠⠠⠠
zayin	ז	ז	⠠⠠⠠	samekh	ס	ס	⠠⠠⠠				
cheth	ח	ח	⠠⠠⠠	ayin	ע	ע	⠠⠠⠠				

Russian Alphabet

The Russian alphabet is sometimes referred to as the *Cyrillic alphabet*.

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
ah	а	А	<i>а</i>	<i>А</i>	⠁
beh	б	Б	<i>б</i>	<i>Б</i>	⠃
veh	в	В	<i>в</i>	<i>В</i>	⠉
gheh	г	Г	<i>г</i>	<i>Г</i>	⠄
deh	д	Д	<i>г or д</i>	<i>Д</i>	⠔
yeh	е	Е	<i>е</i>	<i>Е</i>	⠋
zheh	ж	Ж	<i>ж</i>	<i>Ж</i>	⠵
zeh	з	З	<i>з or ж</i>	<i>З</i>	⠵
ee	и	И	<i>и</i>	<i>И</i>	⠋
kah	к	К	<i>к</i>	<i>К</i>	⠋
ell	л	Л	<i>л</i>	<i>Л</i>	⠋
em	м	М	<i>м</i>	<i>М</i>	⠍
en	н	Н	<i>н</i>	<i>Н</i>	⠎
oh	о	О	<i>о</i>	<i>О</i>	⠋
peh	п	П	<i>п</i>	<i>П</i>	⠎

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
err	р	Р	<i>р</i>	<i>Р</i>	⠋
ess	с	С	<i>с</i>	<i>С</i>	⠉
teh	т	Т	<i>т</i>	<i>Т</i>	⠉
oo	у	У	<i>у</i>	<i>У</i>	⠉
eff	ф	Ф	<i>ф</i>	<i>Ф</i>	⠉
khah	х	Х	<i>х</i>	<i>Х</i>	⠉
tseh	ц	Ц	<i>ц</i>	<i>Ц</i>	⠉
cheh	ч	Ч	<i>ч</i>	<i>Ч</i>	⠉
shah	ш	Ш	<i>ш</i>	<i>Ш</i>	⠉
shchah	щ	Щ	<i>щ</i>	<i>Щ</i>	⠉
yerih	ы	Ы	<i>ы</i>		⠉
eh	э	Э	<i>э</i>	<i>Э</i>	⠉
yu	ю	Ю	<i>ю</i>	<i>Ю</i>	⠉
yah	я	Я	<i>я</i>	<i>Я</i>	⠉

§23. Alphabets:

a. Specific provision is made in this Code for five alphabets — English, German, Greek, Hebrew, and Russian. The letters of the English alphabet are often called *Roman*, and those of the Russian alphabet *Cyrillic*.

b. Some of the letters of the ordinary lower-case Greek alphabet possess an alternative form. The more common ones are:

Name of letter	Sign	Braille equivalent
alpha	α	$\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$
beta	β	$\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$
theta	ϑ	$\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$
sigma	ς	$\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$
phi	φ	$\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$

When these alternative forms occur *instead of* the standard forms throughout a text, the symbols for the standard forms should be used in braille, and the transcriber should call attention to this usage by a transcriber's note. The alternative forms should be used only when the author has assigned distinct meanings to the standard and alternative forms of the same letter.

c. Some Greek letters used in textbooks are obsolete. The more common ones are:

Name of letter	Sign	Braille equivalent
stigma	ς	$\begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix}$
vau	F	$\begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix}$
koph (or qoph)	$\var�$	$\begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix}$
sampi	$\wp$	$\begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix}$

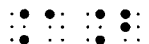
§24. Alphabetic Indicators:

a. Except for the English-letter indicator (see §§26-28), the appropriate alphabetic indicator must be used to specify the alphabet to which a letter belongs. If the letter is lower case, the corresponding alphabetic indicator must precede the letter directly; if the letter is capitalized, so that the capitalization indicator is also required, the alphabetic indicator must precede the capitalization indicator.

- (1) α $\begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix}$
 (Greek lower-case alpha)
- (2) Σ $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$
 (Greek capitalized sigma)

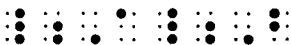
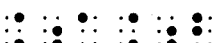
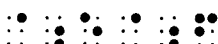
- (3) π 
(Greek lower-case pi)
- (4) ϕ 
(alternative form of Greek lower-case phi)
- (5) ς 
(Greek stigma; obsolete)
- (6) $\mathfrak{a}$ 
(German lower-case ah)
- (7) $\mathfrak{A}$ 
(German capitalized ah)
- (8) $\aleph_0$ 
(Hebrew alef sub zero)
- (9) $\mathfrak{a}$ 
(Russian lower-case ah)
- (10) $\mathfrak{A}$ 
(Russian capitalized ah)

b. The effectiveness of an alphabetic indicator extends only to the letter or, in the case of the English-letter indicator, to a "short-form combination" or a lower-case Roman numeral which follows it. When an alphabetic indicator is required, it must be used with each individual letter of a sequence of letters or, in the case of the English-letter indicator, with the "short-form combination" which follows it. (See §25.)

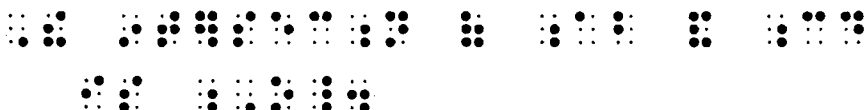
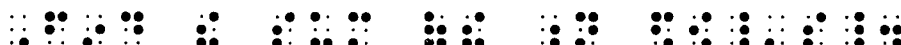
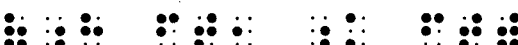
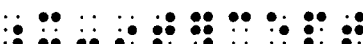
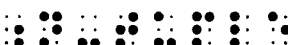
- (1) $\alpha\beta$ 
(Greek lower-case alpha followed by Greek lower-case beta)
- (2) $\mathfrak{A}\alpha + \mathfrak{B}\beta$ 
(German capitalized ah followed by Greek lower-case alpha plus German capitalized beh followed by Greek lower-case beta)
- (3) ab is parallel to cd 
(3) ab is parallel to cd
- (4) iv 
(4) iv

§25. "Single Letters" and "Short-Form Letter Combinations" (See §26):

- a. A "single letter", for the purposes of this Code, must meet the following requirements:

- (1) **AB** 
(boldface capitalized a followed by boldface capitalized b)
- (2) *ab* 
(italic lower-case a followed by italic lower-case b)
- (3) *e* *f* 
(script lower-case e followed by script lower-case f)

b. The English-letter indicator must be used with "single letters" or "short-form combinations" unless specifically prohibited by other rules of this Code.

- (1) $\angle$'s A and B are acute. 
- (2) The intersection of ab and cd is O. 
- (3) Find the sum of the n $\angle$'s. 
(here the shape sign for *angle* is not an omission sign so that the English-letter indicator is required)
- (4) (h ft, k ft) 
(this is not an "enclosed list"; therefore, the English-letter indicator is required with the h and the k)
- (5) (a, 2x, y = z) 
(this is not an "enclosed list"; therefore, the English-letter indicator is required with the a but is not required with the y or z, as would be the case if the parentheses were not present)
- (6) x-intercept 
- (7) n-tuple 
- (8) not-p 
- (9) Exercises A-F 
- (10) Exercise 1-a 
- (11) X-, Y-, and Z-axes. 

(2) $\left| \begin{array}{cc} \frac{d}{du} & \frac{d}{dv} \\ \frac{d}{dx} & \frac{d}{dy} \end{array} \right|$

d. The English-letter indicator must not be used with a "single letter" or "short-form combination" which is an item in an "enclosed list." (For definition of "enclosed list" see §10.)

(1) (0, a, 1, b, 2)

(2) {a, b, c, d}

(3) (ab, cd, ef)

(4) (a, 2x, b)

e. The English-letter indicator must not be used with the letter "s" when this "s" is part of the apostrophe-s combination.

(1) x's, y's, and z's.

f. The English-letter indicator must not be used with a "single letter" or "short-form combination" which is preceded or followed by a comparison sign.

(1) If $a = b$, then $ac = bc$.

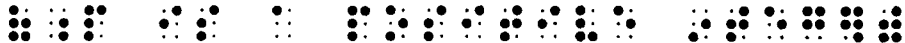
(2) $a = b$, but $c \neq b$.

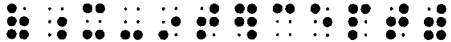
(3) 30% of $N = 63$

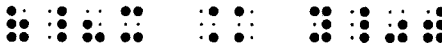
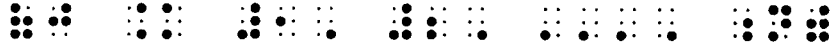
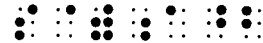
(4) "x = y"

combination of unspaced letters is in direct contact with only its opening or only its closing grouping sign, the English-letter indicator must be used (see §26) or must not be used (see §27) as though the grouping signs were not present. However, if the grouping sign has a prime, subscript or superscript, the English-letter indicator must not be used.

- (1) 1. 
 (a) 
 (b) 

 (2) $|x|$, $[x]$, $\|f\|$       
 (3) $(ab) + (cd)$ 
 (4) $(p \text{ is a positive integer})$

 (5) $(p \text{ and } q)$ 
 (6) $(l, m, n \text{ are in set } R)$

 (7) $(x\text{-intercept})$ 
 (8) $(ab \text{ and } cd)$ 
 (9) $\{x \mid x \text{ has the property } R\}$

 (because of the *such that* sign, which is a comparison sign, the x's do not require the English-letter indicator)
 (10) $(“x = y”)$ 
 (11) Solve for x ($x > y$). 
 (12) $(j = 1, 2, \dots, n)$ 
 (13) $(ab = cd)$ 
 (14) $s]_a^b$ 
 (closing bracket has a subscript and a superscript; therefore, the English-letter indicator is not required with

(closing bracket has a subscript and a superscript; therefore, the English-letter indicator is not required with the s)

b. When only one letter or any combination of unspaced letters has a plural, possessive or ordinal ending, the English-letter indicator must be used (see §26) or must not be used (see §27) as though such endings were not present.

- (1) **x_s** ⠠⠭⠨⠠⠎
 (2) **x's** ⠠⠭⠠⠛⠠⠎
 (3) **nth** ⠠⠋⠠⠋⠠⠋
 (4) **2nth** ⠠⠒⠠⠋⠠⠋⠠⠋

(without the th ordinal ending the English-letter indicator would not be required before the n)

c. A lower-case Roman numeral must be treated as consisting of one letter even when it consists of more than one letter. The English-letter indicator must be used or must not be used in accordance with the rules for any letter (see §§26-28). A capitalized Roman numeral of one letter is subject to these same rules. For capitalized Roman numerals of more than one letter the English-letter indicator must not be used.

§29. **Letters in Diagrams:** When a single English letter in regular type is used as a label in a diagram, the English-letter indicator is required if the letter is in lower case, but must be omitted if the letter is capitalized.

§30. **Letters in Tables:** When letters appear in tables, whether as entries or headings, the English-letter indicator must be used or must not be used in accordance with the rules contained in §§26-28.

RULE V—TYPE FORMS

Type-Form Indicators for Letters, Numerals, and Compound Expressions

Boldface-Type	⠠⠠
Italic-Type	⠠⠠
Sanserif-Type	⠠⠠⠠
Script-Type	⠠⠠

Type-Form Indicators for Words, Phrases, and Mathematical Statements

Opening Boldface-Type	⠠⠠⠠⠠
Opening Italic-Type	⠠⠠⠠⠠
Closing Boldface-Type	⠠⠠⠠⠠
Closing Italic-Type	⠠⠠⠠⠠

(For combinations of capitalization, alphabetic, and type-form indicators, see Appendix, page 208)

§31. **Type Forms:** Specific provision is made in this Code for five type forms — boldface, italic, regular, sanserif, and script. Except for regular type, these type forms must be specified by the appropriate type-form indicator.

§32. **Use of Type-Form Indicators with Letters, Numerals, and Compound Expressions:**

a. Subject to the provisions of §34, the appropriate type-form indicator must be used to express the type form of a letter. The type-form indicator for a letter must always be followed by an alphabetic indicator.

- (1) *a* ⠠⠠⠠⠠⠠⠠⠠
(italic English lower-case a)
- (2) *A* ⠠⠠⠠⠠⠠⠠⠠
(italic English capitalized a)
- (3) *α* ⠠⠠⠠⠠⠠⠠⠠
(italic German lower-case ah)
- (4) **a** ⠠⠠⠠⠠⠠⠠⠠
(boldface English lower-case a)
- (5) **A** ⠠⠠⠠⠠⠠⠠⠠
(boldface English capitalized a)
- (6) **α** ⠠⠠⠠⠠⠠⠠⠠
(boldface Greek lower-case alpha)
- (7) **ɑ** ⠠⠠⠠⠠⠠⠠⠠
(boldface German lower-case ah)
- (8) **а** ⠠⠠⠠⠠⠠⠠⠠
(boldface Russian lower-case ah)
- (9) *ɑ* ⠠⠠⠠⠠⠠⠠⠠
(script English lower-case a)
- (10) *A* ⠠⠠⠠⠠⠠⠠⠠
(script English capitalized a)
- (11) *α* ⠠⠠⠠⠠⠠⠠⠠
(script German lower-case ah)
- (12) *א* ⠠⠠⠠⠠⠠⠠⠠
(script Hebrew alef)

- (13) h ⠠⠠⠠⠠⠠⠠
 (sanserif English lower-case h)
- (14) H ⠠⠠⠠⠠⠠⠠⠠
 (sanserif English capitalized h)
- (15) *abcd* ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
 (regular a, italic b, boldface c, script d)

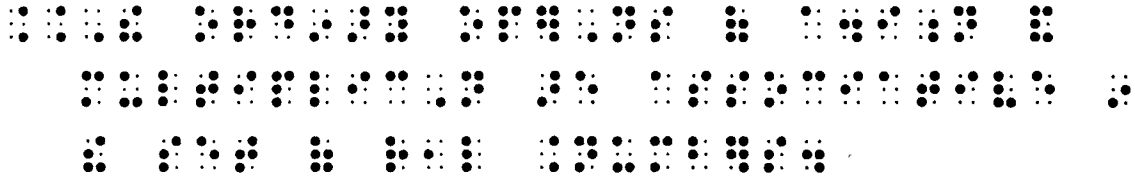
b. Subject to the provisions of §34, the appropriate type-form indicator must be used to express the type form of a numeral. The type-form indicator for a numeral must always be followed by the numeric indicator. If a numeral to be transcribed by using type-form indicators contains more than one digit, and is all of one type form, the type-form indicator and the numeric indicator must be used only before the first digit. If there is a transition from one type form to another non-regular type form within the same numeral, the new type-form indicator followed by the numeric indicator must be used before the first digit of the new type form. If the transition is to regular type, only the numeric indicator must be used.

- (1) 0 ⠠⠠⠠⠠
 (boldface zero)
- (2) 2 ⠠⠠⠠
 (script 2)
- (3) 345 ⠠⠠⠠⠠⠠⠠
 (345 in boldface type)
- (4) 3.5 ⠠⠠⠠⠠⠠⠠
 (3.5 in italic type)
- (5) 345 ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
 (italic 3, boldface 4, script 5)
- (6) 485 ⠠⠠⠠⠠⠠⠠⠠
 (boldface 4, regular 3 and 5)

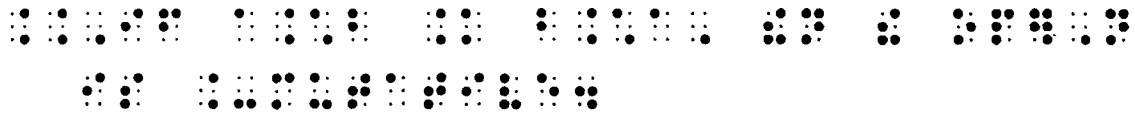
c. Subject to the provisions of §34, when a numeral is joined to a word or an abbreviation by a hyphen and the whole expression is printed in non-regular type, the appropriate type-form indicator must be used before the numeral only, but affects the entire compound expression. If there is a change in type form after the hyphen to regular type, the hyphen must be preceded by the literary termination symbol ⠠⠠⠠ (dots 6, 3). If there is a change in type form after the hyphen to non-regular type, only the appropriate type-form indicator must be used after the hyphen.

- (1) 45-ohm ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
 (the whole expression is in italic type)

- (4) *The ordinary operations of addition and multiplication are associative in the set of real numbers.*



- (5) *If $a \circ b = b \circ a$, then the operation is commutative.*



§34. Non-Use of Type-Form Indicators:

- a. A type-form indicator must not be used when a letter or a numeral is printed in regular type.

b. When any material, mathematical or literary, is printed in non-regular type that has no mathematical significance, the variant type form must not be represented in the transcription. Frequently, it is the practice to print the letters of all formulas throughout a book in italicized type. This practice must not be carried over to the transcription unless the author has specifically distinguished between two meanings of the same letter, assigning one meaning to the letter in regular type and another to the letter in italic type. In addition, a variant type form is often used, particularly at the lower grade levels, for the sole purpose of attracting the reader's attention. Such variant type forms must also not be represented in the transcription.

§35. Boldface Type:

a. When certain signs of operation or comparison are printed in boldface type, this Code employs the device of placing dots 4-5-6 before the corresponding symbol. The specific signs to which this technique applies are listed in appropriate sections throughout the Code, and the transcriber must not use this technique with any other sign. When used in this way, dots 4-5-6 must not be regarded as the boldface type-form indicator but as an integral part of the symbol to which it belongs. This technique has been used only when the distinction between the regular and boldface forms of the same sign has mathematical significance. Dots 4-5-6 are also used as part of the technique for representing filled-in shapes (see §108).

b. Boldface type, used in many texts to identify letters as vectors, must be preserved in the transcription. When both boldface type and arrows of uniform construction are used in conjunction to represent vectors, the arrows themselves must be omitted from the transcription unless the author calls special attention to them as a notational device, but a transcriber's note must be included indicating their presence in the ink print copy.

RULE VI—PUNCTUATION SIGNS AND SYMBOLS

Punctuation Indicator ⠠

Punctuation Marks

Apostrophe ’ ⠨

Colon : ⠆

Comma

Literary	,	⠠⠨
Mathematical	,	⠠⠨⠠

Dash

Short	—	⠠⠠⠠⠠
Long	—	⠠⠠⠠⠠⠠⠠

Ellipsis	...	⠠⠠⠠⠠
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Exclamation Point	!	⠠⠠⠠
-------------------	---	-----

Hyphen	-	⠠⠠
--------	---	----

Period	.	⠠⠠⠠
--------	---	-----

Question Mark	?	⠠⠠⠠
---------------	---	-----

Quotation Marks

Left inner	'	⠠⠠⠠⠠
------------	---	------

Left outer	"	⠠⠠
------------	---	----

Right inner	'	⠠⠠⠠⠠
-------------	---	------

Right outer	"	⠠⠠
-------------	---	----

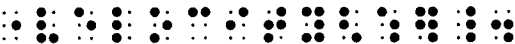
Semicolon	;	⠠⠠⠠
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§36. **Modes of Punctuation:** Since numerals are represented by symbols in the lower part of the cell, and since these symbols also serve as punctuation marks, it is necessary to formulate rules concerning punctuation so that the meanings of such symbols are unambiguous. This Code employs two modes of punctuation — mathematical and literary.

§37. **Use of the Punctuation Indicator:** Subject to the provisions of §38, the punctuation indicator must be used *before* a punctuation mark and *after* any symbol of the type listed below. In all these circumstances, the mode of punctuation is considered to be mathematical.

i. After any braille indicator.

(1)	$\frac{1}{2}$	$\frac{3}{4}$	⠠⠠⠠⠠⠠⠠⠠⠠	⠠⠠⠠⠠⠠⠠⠠⠠
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- (2) velocity. 
(bar over "velocity")

ii. After any numeric symbol written as in the Nemeth Code.

- (1) 0. 

- (2) "49" 

iii. After a Roman numeral.

- (1) I, II, III. 

iv. After a dash or ellipsis, when these occur in a mathematical context. When the nature of the context is in doubt, the punctuation indicator must be used.

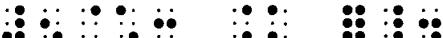
- (1) $24 = 6 + \text{---}$. 

- (2) 1, 3, ... 

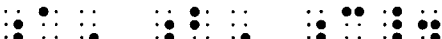
v. After any reference symbol.

- (1) note*. 

vi. After the general omission symbol.

- (1) $5 \times 3 = ?$. 

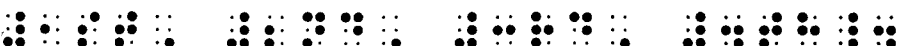
vii. After a "single letter."

- (1) a, b, c. 

viii. After a sequence of more than one letter in which each letter has a separate identity, provided that such a sequence is not an abbreviation.

- (1) $\triangle ABC$. 

ix. After ordinal, plural, or possessive endings which are joined to numerals, letters, or other mathematical expressions.

- (1) 1st, 2nd, 3rd, 4th. 

- (2) Mary, Sally,
- (3) a, ar, ar²,
- (4)
$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

b. The ellipsis is subject to the same spacing rules as the long dash. See §42.

- (1) $x + y + \dots$
(the period applies to the ellipsis)
- (2) 1, 3, 5, ..., 15.
(the comma applies to the ellipsis)
- (3) $p_1^{\alpha_1} \dots p_r^{\alpha_r}$
(the base-line indicator applies to the ellipsis)
- (4) $(\dots, -1, 0, 1, \dots)$
(the opening and closing grouping symbols apply to their respective ellipses)
- (5) $12¢ + 14¢ = \dots¢$
(the cent sign applies to the ellipsis)

§44. **Exclamation Point:** The exclamation point is represented by the same sign of ink print as the factorial sign. The context is usually sufficiently clear in regard to this distinction so that the possibility of doubt in choosing the proper symbol is small.

§45. **Hyphen:** The hyphen is represented by the same sign of ink print as the minus sign. Since the corresponding braille symbols also coincide, a minimum of decision-making in this regard is required of the transcriber. A space must be left between a hyphen and an adjacent dash.

v. Any sequence of more than one letter in which each letter has a separate identity.

(1) xy sine z

vi. Any modifier symbol.

(1) $\xrightarrow{\text{heat}}$

vii. The radical symbol.

(1) $\sqrt{\text{four}}$

viii. Any operation symbol.

(1) nine — seven = two

(2) ergs/cm³

(3) 60 min./hour

(4) statvolt-cm/statamp-oersted

(5) $\frac{\text{distance}}{\text{time}} = \text{rate}$

(6) seven + three

(7) people who travel by bus/people who travel by car

ix. Any comparison symbol, even though there is a space between it and the word, part word, or abbreviation.

(1) 1 hour = 60 minutes

(2) Let $3x =$ the larger number

(3) seven + three = ten

- (4) Copy and replace $\square$ by $=$ or $\neq$ to make a true sentence.

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} \neq \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} \neq \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} \neq \frac{1}{4}$
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- (5) It is a fundamental principle that $=$'s added to $=$'s are $=$.

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
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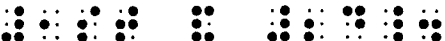
b. Contractions must not be used in abbreviated function names in any context. In addition, contractions must not be used in unabbreviated function names which appear in a mathematical context. In particular, the word "arc" must not be contracted when immediately preceded or followed by mathematical symbols, whether spaced or unspaced.

- (1) $\sin x$ $\sin x$
- (2) $\cosh x$ $\cosh x$
- (3) $\sin x + \sin y$ $\sin x + \sin y$
- (4) $\cos \left(\arctan x + \frac{\pi}{3} \right)$ $\cos \left(\arctan x + \frac{\pi}{3} \right)$
- (5) $2 \arcsin x$ $2 \arcsin x$
- (6) Arc Sine x Arc Sine x
- (7) Arc ACB is a major arc. Arc ACB is a major arc.

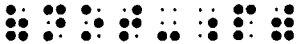
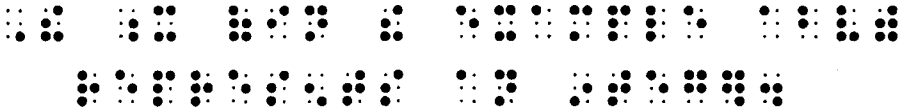
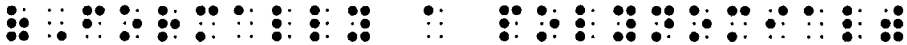
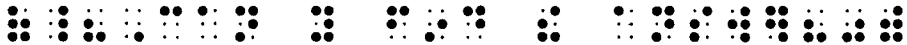
c. The contractions for *to*, *into*, and *by* must not be used before any of the items listed below. When the contraction for *into* may not be used, the contraction for "in" may nevertheless be used in "into" unless otherwise prohibited.

- i. Before any of the items in a above.

- (1) From a to z. From a to z.
(contraction not used according to a(i))
- (2) From $\frac{1}{4}$ to $\frac{1}{2}$. From $\frac{1}{4}$ to $\frac{1}{2}$.
(contraction not used according to a(i))

- (3) 2nth 
- (4) 1st and 2d. 

e. The one-cell whole word alphabet contractions for *but*, *can*, ..., *you*, *as* and the one-cell lower-sign whole-word contractions for *be*, *enough*, *were*, *his*, *in*, *was*, whether capitalized, italicized, or neither, must not be used when these words are in direct contact with any grouping symbol. The contractions, whole-word or part-word, for *and*, *for*, *of*, *the*, *with*, whether capitalized, italicized, or neither, must also not be used when in direct contact with any grouping symbol. If any punctuation intervenes between a grouping symbol and any contraction of the types mentioned above, the rule still applies. When this rule precludes the use of a contraction in one part of a word, no part of the word may be contracted.

- (1) (and, in addition) 
- (2) (that is) 
- (3) (not-p) 
- (4) The x (in the example above) represents an integer.

- (5) (of course) 
- (6) (Formally a polynomial)

- (7) (officially withdrawn) 
- (8) ("Can you find the answer?")

- (9) ("in addition, $x \neq 0$ ")

- (10) ("Of course not!") 
- (11) (Give the command) 

f. Contractions must not be used when they are likely to be mistaken for mathematical expressions.

- (1) Use the $\int$ to find the volume.


- (2) Can $C = 100$?
 (3) $a = b$, but $b \neq c$.
 (4) We see that $c = d$.
 (5) Let *this* be an angle in standard position.

("this" is spelled out because in immediately surrounding text in ink print the Greek letter "theta" (θ) also appeared)

§56. Use of Contractions and Short-Form Words: Subject to the conditions of §55, the use of contractions and short-form words of English Braille must be used.

- (1) 1 light-year
 (2) not-p
 (3) x-intercept
 (4) unary, binary, ..., m-ary.
 (5) $\frac{1}{2}$ -off sale
 (6) 9-inch
 (7) hydrogen-3
 (8) 360° -interval
 (9) ?-ounce
 (the omission symbol replaces a question mark in ink print)
 (10) (ft.-pound)
 (11) inch-pound²
 (12) 60 min./hour
 (13) (rate) $\times$ (time) = (distance)

(6) $9 - 5 = -?-$



(a question mark is preceded and followed by a hyphen in ink print)

(7) $(5,) + (, 15) = (7, 13)$



(blank spaces occur in ink print)

(8) $5 \times 25 =$



(a blank space occurs in ink print)

(9) $\text{five} \times \text{—} = \text{fifteen}$



(a dash occurs in ink print)

(10) $2, 4, 6, \dots, 10.$



(an ellipsis occurs in ink print)

(11) $\text{The quick brown fox} \dots$



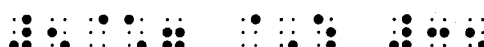
(an ellipsis occurs in ink print)

(12) $\square + \circ = 5$



(a square and circle occur in ink print)

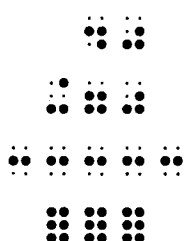
(13) $5 \times 7 \sim 35$



(an extended tilde occurs in ink print)

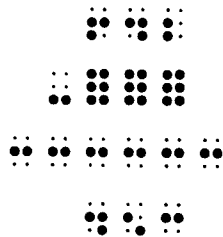
§58. Omissions in Work Arranged Spatially for Computation: In work arranged spatially for computation, only the general omission symbol may be used in braille regardless of how the omission is denoted in ink print. In addition, the number of general omission symbols to be used must be the same as the number of omission signs which occur in ink print.

(1)
$$\begin{array}{r} 40 \\ +70 \\ \hline ??? \end{array}$$



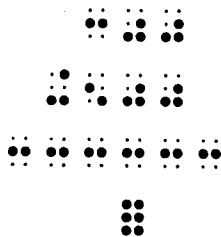
(question marks are shown in ink print)

(2)
$$\begin{array}{r} 642 \\ - ??? \\ \hline 458 \end{array}$$



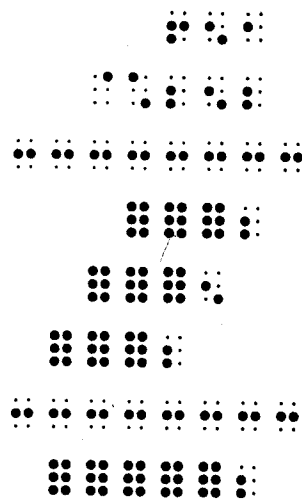
(question marks are shown in ink print)

(3)
$$\begin{array}{r} 300 \\ + 500 \\ \hline ? \end{array}$$



(a question mark is shown in ink print)

(4)
$$\begin{array}{r} 651 \\ \times 252 \\ \hline \dots 2 \\ \dots 5 \\ \dots 2 \\ \hline \dots 2 \end{array}$$



(dots are shown in ink print)

(5)
$$\begin{array}{r} 144 \\ 6 \overline{) 864} \\ \underline{6} \\ 26 \\ \underline{24} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

(dots are shown in ink print)

§59. **Spacing with Omissions:** The general omission symbol should be spaced in the same manner as the material which it replaces. Other omission symbols must be spaced in accordance with the rules governing the spacing of those symbols.

RULE XI—CANCELLATION

Cancellation Indicators

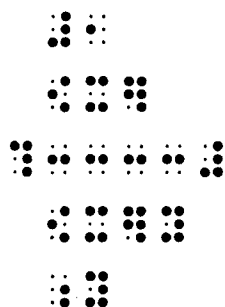
Opening ⠠

Closing ⠨

§60. **Cancellation Indicators:** The cancellation indicators must be used to show the extent of a mathematical expression which has been canceled in ink print. A spatial arrangement must be used when cancellation is represented in braille. Whenever a fraction or any of its parts is canceled, a spatial arrangement must be used for that fraction. Items which are individually canceled in ink print must be represented as individually canceled in the transcription.

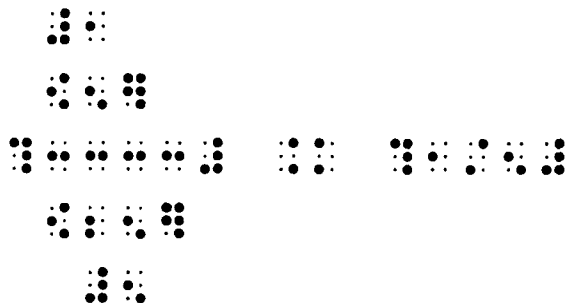
(1)

$$\frac{\cancel{xy}}{y}$$



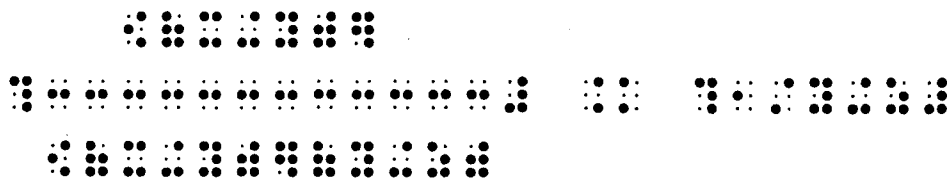
(2)

$$\frac{\cancel{5}}{\cancel{25}} = \frac{1}{5}$$



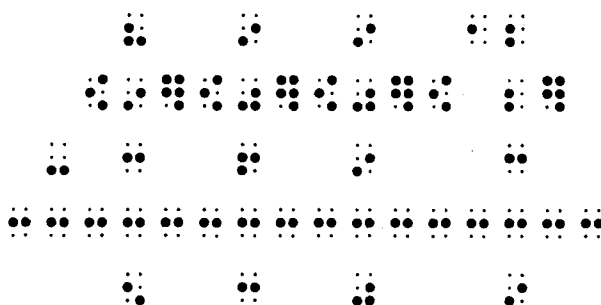
(3)

$$\frac{\cancel{-(x+y)}}{\cancel{-(x+y)}(y+z)} = \frac{1}{y+z}$$



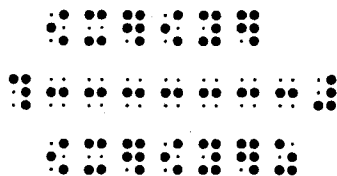
(4)

$$\begin{array}{r} 8 9 9 12 \\ \cancel{8} \cancel{8} \cancel{8} \cancel{2} \\ - 3 6 9 3 \\ \hline 5 3 0 9 \end{array}$$



(5)

$$\frac{\cancel{xy}}{\cancel{xyz}}$$



RULE XII—FRACTIONS

Fraction Indicators

Simple

Opening $\frac{\cdot}{\cdot}$

Closing $\frac{\cdot}{\cdot}$

Complex

Opening $\frac{\cdot}{\cdot} \frac{\cdot}{\cdot}$

Closing $\frac{\cdot}{\cdot} \frac{\cdot}{\cdot}$

Hypercomplex

Opening $\frac{\cdot}{\cdot} \frac{\cdot}{\cdot} \frac{\cdot}{\cdot}$

Closing $\frac{\cdot}{\cdot} \frac{\cdot}{\cdot} \frac{\cdot}{\cdot}$

Fractional Part of a Mixed Number

Opening $\frac{\cdot}{\cdot} \frac{\cdot}{\cdot}$

Closing $\frac{\cdot}{\cdot} \frac{\cdot}{\cdot}$

Fraction Lines

Used with Simple-Fraction Indicators

Diagonal line or slash $\frac{\cdot}{\cdot}$

Horizontal $\frac{\cdot}{\cdot}$

Used with the Fractional Part of a Mixed Number

Diagonal line or slash $\frac{\cdot}{\cdot} \frac{\cdot}{\cdot}$

Horizontal $\frac{\cdot}{\cdot} \frac{\cdot}{\cdot}$

Used with Complex-Fraction Indicators

Diagonal line or slash $\frac{\cdot}{\cdot} \frac{\cdot}{\cdot} \frac{\cdot}{\cdot}$

Horizontal $\frac{\cdot}{\cdot} \frac{\cdot}{\cdot} \frac{\cdot}{\cdot}$

Used with Hypercomplex-Fraction Indicators

Horizontal ——— $\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$

Used with a Spatial Arrangement

Horizontal (varying in length) ——— $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$

§61. Simple Fractions: For the purposes of this Code, a *simple fraction* is one whose numerator and denominator contain no fractions except possibly at the superscript or subscript level.

§62. Use of Simple-Fraction Indicators:

a. Simple-fraction indicators must be used, except in the case of mixed numbers, to enclose a simple fraction whose numerator and denominator are separated by a horizontal fraction line in ink print.

- (1) $\frac{1}{8}$ $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$
- (2) $x^{\frac{1}{2}}$ $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$
- (3) $\frac{a+b}{c}$ $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$
- (4) $\frac{x^{\frac{1}{2}}}{2}$ $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$
- (5) $\text{rate} = \frac{\text{distance}}{\text{time}}$ $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$

b. Simple-fraction indicators must be used to enclose a simple fraction whose numerator and denominator are separated by a diagonal line in ink print, when the expressions on either side of the diagonal line appear at different levels relative to it, or in different type size than is normal for the purpose for which these expressions are used.

- (1) $a + \frac{b}{c} + d$ $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$
(in ink print, the numerator is written near the top of the diagonal line and the denominator is written near the bottom)
- (2) $3x/y$ $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$
(in ink print the 3, x, and y are at the same level, but the x and y are in smaller type than the 3)

§63. Non-Use of Simple-Fraction Indicators:

a. Simple-fraction indicators must not be used to enclose the fractional part of a mixed number.

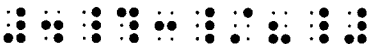
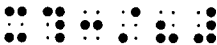
- (1) $4\frac{3}{8}$ $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$
- (2) $2\frac{3}{4}x$ $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$

b. Simple-fraction indicators must not be used to enclose a simple fraction whose numerator and denominator are separated by a diagonal line in ink print when the expressions on either side of the diagonal line appear at the same level relative to it, or are of the

same type size as the surrounding mathematical text. Sometimes the expressions on either side of the diagonal line are not the terms of a fraction at all. Even when they are, the transcriber cannot always be certain of where the fraction begins or ends. Accordingly, it is better to avoid the use of indicators altogether in these cases and permit the braille reader to make a judgment based on the same information that is available to the sighted reader.

- (1) $1/3$ 
(in ink print, 1 and 3 are at the same level)
- (2) $x^{1/2}$ 
(in ink print, 1 and 2 are at the same level; although the 1 and 2 are in smaller type, they are of normal size for printing superscripts)
- (3) $x^{1/2}$ 
(in ink print, the x and 2 are at the same level and are of normal size for printing base-line signs)
- (4) $x^{1/2/7}$ 
(in ink print, 1 and 2 are at the same level and x and 7 are at the same level; each pair of signs is of normal size for printing at its respective level)
- (5) $a + b/c + d$ 
(in ink print, all letters are of normal size and at the same level on either side of a diagonal line)
- (6) $(a + b)/(c + d)$ 
(in ink print, all letters are of normal size and at the same level on either side of a diagonal line)
- (7) $I/(n)$ 
(the expressions on either side of the diagonal line are not the terms of a fraction)
- (8) $1/31/70$ 
(the expression represents a date)

§64. **Mixed Numbers:** For the purposes of this Code, a mixed number is an expression which begins with a numeral and is followed, usually in smaller type, by a simple fraction whose numerator and denominator are both numerals. The fraction line of this simple fraction may be either horizontal or diagonal in ink print. The mixed-number indicators must be used to enclose the fractional part of a mixed number. An expression is not a mixed number if it contains any letter, even though such an expression is of the same form as a mixed number in every other respect.

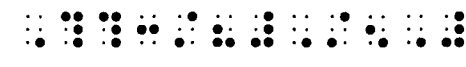
- (1) $4\frac{3}{8}$ 
(2) $4\frac{3}{8}$ 
(3) $x\frac{3}{8}$ 
(this is not a mixed number; fraction in smaller type than the x)
- (4) $x\frac{3}{8}$ 
(this is not a mixed number; fraction in smaller type than the x)

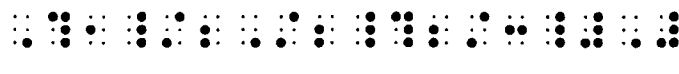
(5) $3 \frac{1}{y}$ 

(this is not a mixed number; fraction in smaller type than the 3)

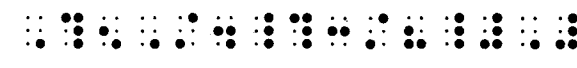
§65. Complex Fractions: For the purposes of this Code, a complex fraction is one whose numerator, denominator, or both, contains at least one simple fraction. A fraction is not a complex fraction if the only simple fractions it contains are at the superscript or subscript level.

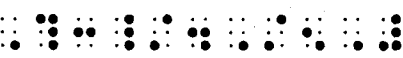
§66. Use of Complex-Fraction Indicators: Complex-fraction indicators must be used to enclose a complex fraction.

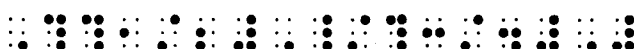
(1) $\frac{\frac{3}{8}}{5}$ 

(2) $\frac{1/2}{2\frac{2}{3}}$ 

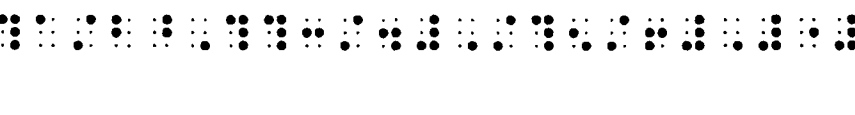
(3) $\frac{2/3}{3/2}$ 

(4) $\frac{5}{4\frac{3}{8}}$ 

(5) $\frac{3/4}{5}$ 

(6) $\frac{1}{2} / \frac{3}{4}$ 

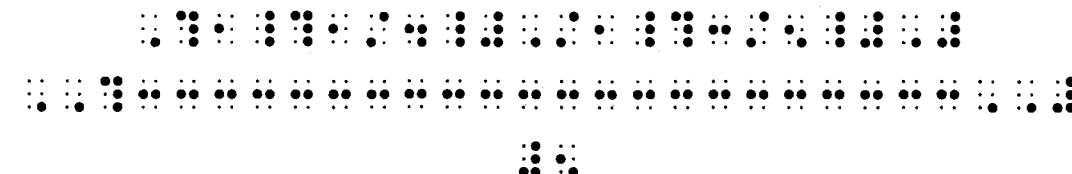
§67. Hypercomplex Fractions: For the purposes of this Code, a hypercomplex fraction is one whose numerator, denominator, or both, contain at least one complex fraction. A fraction is not a hypercomplex fraction if the only complex fractions it contains are at the superscript or subscript level.

(1) $\frac{a}{\frac{\frac{3}{4}}{b^{\frac{5}{6}}}}$ 

(this is not a hypercomplex fraction)

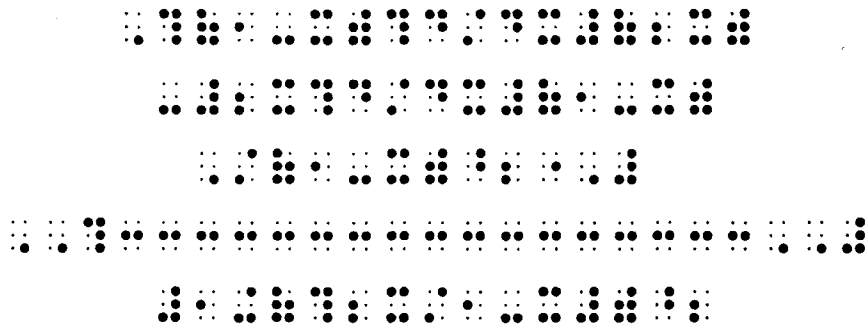
§68. Use of Hypercomplex-Fraction Indicators:

a. Hypercomplex-fraction indicators must be used to enclose a hypercomplex fraction. The use of a linear arrangement within a spatial arrangement is preferable to an arrangement which is entirely linear or entirely spatial.

(1) $\frac{1\frac{1}{4}}{\frac{1\frac{3}{5}}{5}}$ 

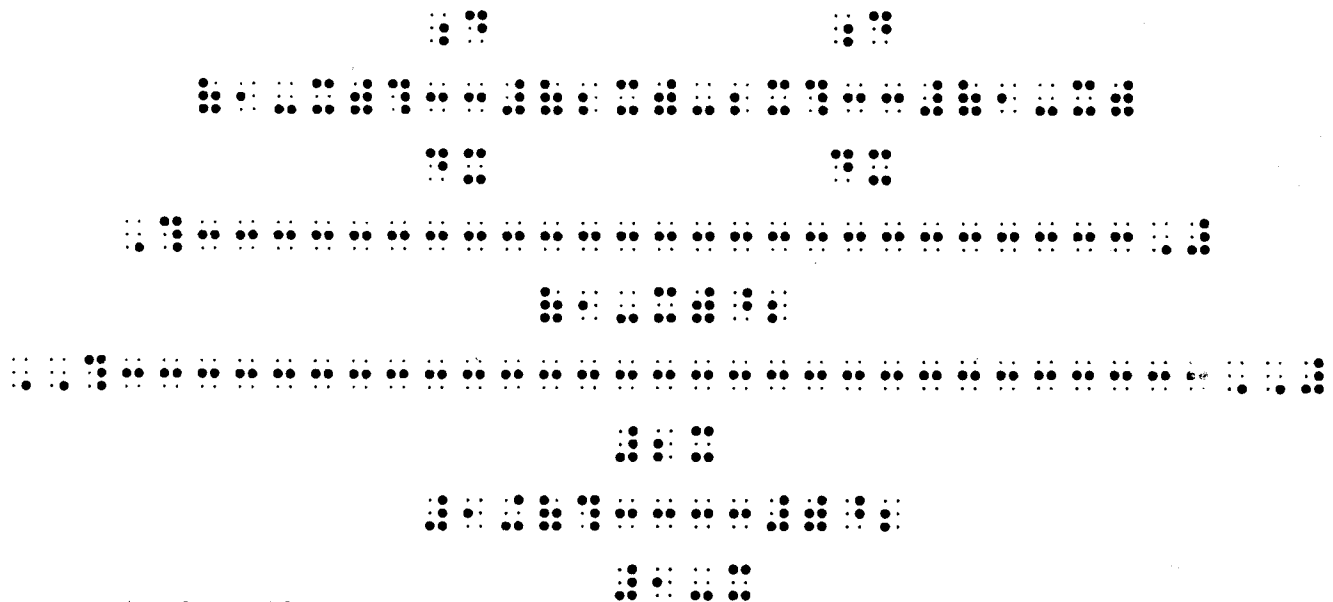
(preferred method of transcribing a hypercomplex fraction)

$$(2) \quad \frac{(1-x) \frac{d}{dx} (2x) - 2x \frac{d}{dx} (1-x)}{(1-x)^2} \\ 1 + \left(\frac{2x}{1-x} \right)^2$$



(preferred method of transcribing a hypercomplex fraction)

$$(3) \quad \frac{(1-x) \frac{d}{dx} (2x) - 2x \frac{d}{dx} (1-x)}{(1-x)^2} \\ 1 + \left(\frac{2x}{1-x} \right)^2$$

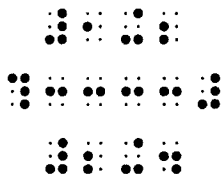


(complete spatial arrangement)

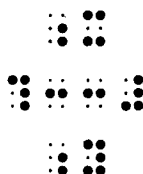
must be centered on their fraction lines. The runover of an expression which is too long to be centered on the fraction line which applies to it may be effected at suitable places in accordance with the rules for runovers, but each portion of the divided expression must be centered on the fraction line to which the expression, as a whole, applies.

b. In general, the linear arrangement for fractions must be used when not expressly forbidden in the case of continued fractions. However, when fractional notation is first presented to the reader, as in the lower grades, or when there is any other special need, any fraction may be represented spatially.

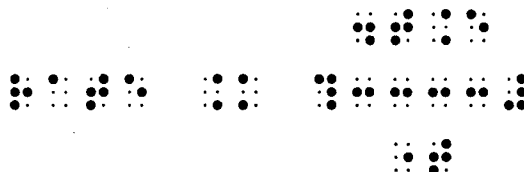
(1) $\frac{1+2}{2+4}$



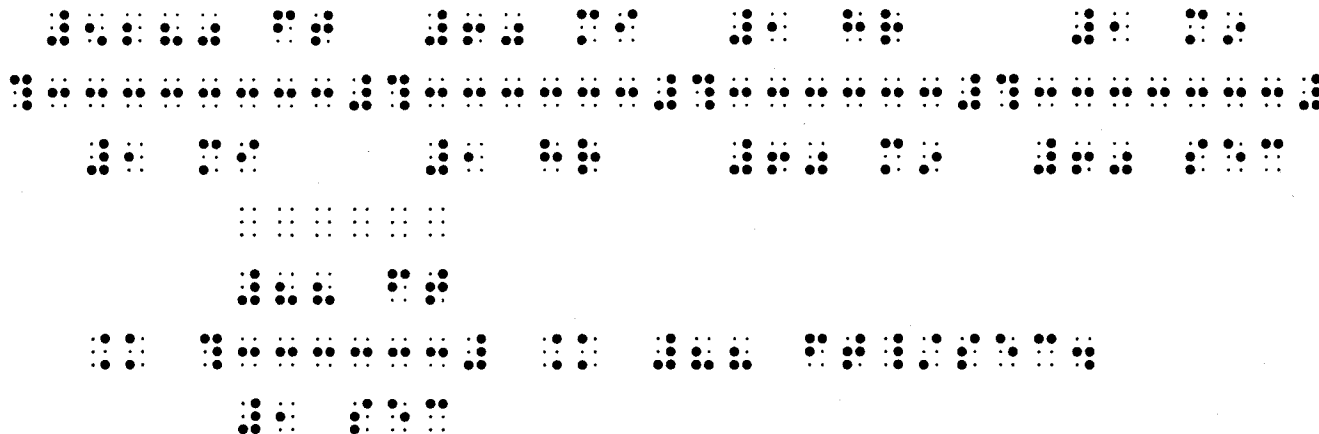
(2) $\frac{x}{y}$



(3) $\text{rate} = \frac{\text{distance}}{\text{time}}$



(4) $\frac{5280 \text{ ft}}{1 \text{ mi}} \frac{60 \text{ mi}}{1 \text{ hr}} \frac{1 \text{ hr}}{60 \text{ min}} \frac{1 \text{ min}}{60 \text{ sec}} = \frac{88 \text{ ft}}{1 \text{ sec}} = 88 \text{ ft/sec.}$



c. For spatial arrangement of fractions in connection with cancellation see §60.

d. For spatial arrangement of hypercomplex fractions see §68.

e. For spatial arrangement of continued fractions see §69.

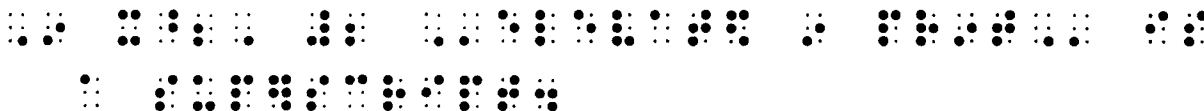
RULE XIII—SUPERSCRIPTS AND SUBSCRIPTS

Level Indicators

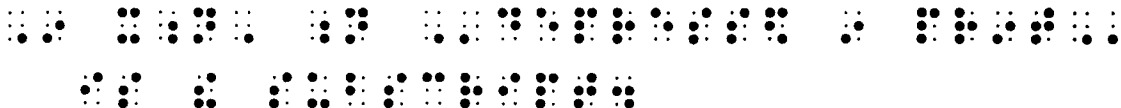
Base Line	⠠
Superscript	⠠
Superscript with Superscript	⠠ ⠠
Superscript with Subscript	⠠ ⠠
Superscript with Superscript with Superscript	⠠ ⠠ ⠠
Superscript with Superscript with Subscript	⠠ ⠠ ⠠
Superscript with Subscript with Superscript	⠠ ⠠ ⠠
Superscript with Subscript with Subscript	⠠ ⠠ ⠠
Subscript	⠡
Subscript with Superscript	⠡ ⠠
Subscript with Subscript	⠡ ⠠
Subscript with Superscript with Superscript	⠡ ⠠ ⠠
Subscript with Superscript with Subscript	⠡ ⠠ ⠠
Subscript with Subscript with Superscript	⠡ ⠠ ⠠
Subscript with Subscript with Subscript	⠡ ⠠ ⠠
Contraction for Comma and Optional Space at Superscript or Subscript Level	⠠

§71. **Nature of Superscripts and Subscripts:** It is characteristic of mathematical expressions to employ signs, usually in smaller type, which are elevated or depressed relative to the *base line*. A sign which is elevated relative to the base line is called a *superscript*; one which is depressed relative to the base line is called a *subscript*. When an entire expression is at the superscript or subscript level, it should be written without an indicator in braille, but its position must be explained to the reader by a transcriber's note.

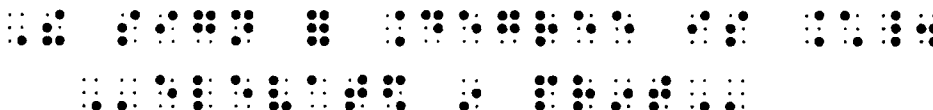
- (1) In x^2 , ² is a superscript.



- (2) In x_n , _n is the subscript.



- (3) The sign for *degree* is °.



§72. **Hierarchy of Superscripts and Subscripts:** Superscripts or subscripts may carry superscripts or subscripts of their own; the latter are then referred to as superscripts or subscripts of *second order*, and are thus distinguished from the former, which are called superscripts or subscripts of *first order*. Second order superscripts or subscripts may, in turn, carry superscripts or subscripts of their own, which are then called superscripts or subscripts of *third order*. While it is theoretically possible for a superscript or subscript to be of order higher than the third, this situation rarely arises in practice.

§73. **Level Indicators:** A level indicator other than the base-line indicator identifies the symbols which follow it as representing a superscript or subscript. The *base-line* indicator identifies the symbols which follow it as representing signs on the base line. The *degree* of elevation or depression specified by a level indicator is *always with respect to the base line*; the symbol which precedes the indicator, if it represents a sign at some other level, plays no role in this regard.

§74. **Orientation by Level Indicator:**

a. The effect of a level indicator with one component is to direct the reader's attention upward or downward from the base line according as that component is the superscript or the subscript indicator.

- (1) x^2 ⠠⠭⠨⠼⠼

(x squared)

- (2) y^3 ⠠⠽⠨⠼⠼

(y cubed)

- (3) x^* ⠠⠭⠨⠼⠼

(x carries an asterisk as a superscript)

- (4) x^{-2} ⠠⠭⠨⠼⠼

(x carries minus 2 as a superscript)

(5) x_a $\begin{array}{ccc} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{array}$

(x sub a)

(6) x_{-2} $\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array}$

(x carries minus 2 as a subscript)

b. The effect of a level indicator with two components may be analyzed as follows:

i. The first component directs the reader's attention upward or downward from the base line according as that component is, in itself, the superscript or subscript indicator.

ii. The second component then directs the reader's attention upward or downward from this new position according as the second component is, in itself, the superscript or subscript indicator.

(1) n^{xy} $\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array}$

(n carries a superscript x which carries a superscript y)

(2) x^{na} $\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array}$

(x carries a superscript n which carries a subscript a)

(3) x_n^a $\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array}$

(x carries a subscript n which carries a superscript a)

(4) n_{xy} $\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array}$

(n carries a subscript x which carries a subscript y)

c. The effect of a level indicator with three components may be analyzed as follows:

i. The first two components direct the reader's attention from the base line to the position described in b above.

ii. The third component then directs the reader's attention upward or downward from this new position, according as that component is, in itself, the superscript or the subscript indicator.

(1) n^{xyz} $\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array}$

(n carries a superscript x which carries a superscript y which carries a superscript z)

(2) $n^{xyz \dots}$ $\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array}$

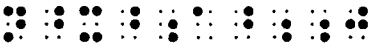
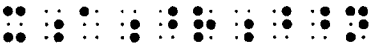
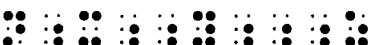
(the ellipsis indicates the presence of superscripts of increasingly higher order; the dots are printed obliquely)

(3) x^{yz_a} $\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array}$

(x carries a superscript y which carries a superscript z which carries a subscript a)

(4) $x^y_{a^n}$ $\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array}$

(x carries a superscript y which carries a subscript a which carries a superscript n)

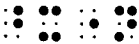
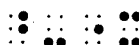
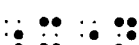
- (5) $n^{x_{aj}}$ 
 (n carries a superscript x which carries a subscript a which carries a subscript j)
- (6) x_{a^n} 
 (x carries a subscript a which carries a superscript n which carries a superscript n)
- (7) $x_{a^{nb}}$ 
 (x carries a subscript a which carries a superscript n which carries a subscript b)
- (8) $x_{p_{am}}$ 
 (x carries a subscript p which carries a subscript a which carries a superscript m)
- (9) n_{xyz} 
 (n carries a subscript x which carries a subscript y which carries a subscript z)
- (10) $n_{xyz...}$ 
 (the ellipsis indicates the presence of subscripts of increasingly higher order; the dots are printed obliquely)

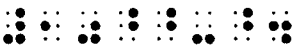
d. The effect of a level indicator with more than three components may be analyzed in the same manner suggested for level indicators with two or three components.

§75. **Left Superscripts and Subscripts:** A superscript or subscript may occupy a position to the left, as well as to the right, of the sign to which it applies. The words *left* or *right* are then used with the words *superscript* or *subscript* to make the distinction in position.

A right or left superscript or subscript is represented as such merely by preserving the relative horizontal positions of the superscript or subscript symbol and the symbol to which it applies. Each must be preceded by its appropriate level indicator.

Left superscripts or subscripts of the third or higher order, although rare, are treated in the manner suggested by the examples below.

- (1) $^x n$ 
 (x is a left superscript to n)
- (2) $^{-x}$ 
 (the minus sign is a left superscript to x)
- (3) $_x n$ 
 (x is a left subscript to n)
- (4) $_x n_y$ 
 (x is a left subscript to n, y is a right subscript to n)

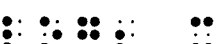
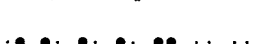
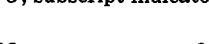
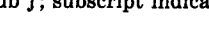
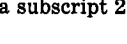
- (5) 10^{-4} 
 (10 to the minus 4 power; the minus is superscript to the 4)
- (6) $n_a x$ 
 (n sub a is a left superscript to x)
- (7) $a^n x$ 
 (a is a left subscript to n, the combination is a left superscript to x)
- (8) $n^a x$ 
 (n to the a power is a left subscript to x)
- (9) a_n^x 
 (a is a left superscript to n, the combination is a left subscript to x)
- (10) x_y^n 
 (x sub y is a left subscript to n)
- (11) y_x^n 
 (y is a left subscript to x, the combination is a left subscript to n)
- (12) $p^{bc} x$ 
 (p carries a right superscript b and c is a left superscript to x)

§76. **Direct Superscripts and Subscripts:** A superscript or subscript which occupies, respectively, a position directly over or under the sign to which it applies is called a *modifier* (see Rule XIV).

§77. **Numeric Subscripts:** The subscript indicator must not be used to indicate a numeric subscript provided that all of the following conditions hold:

- i. The corresponding numeric sign must be a *right*, and not a left, subscript.
- ii. The corresponding numeric sign must be a subscript of *first order*, and not of higher order.
- iii. The sign with which the numeric subscript is associated must be an abbreviated function name or a letter which has a separate identity. In the latter case, this letter must not be any letter which represents a numeral in a non-decimal base. Otherwise, the letter may be from any alphabet and in any type form, and may be modified by one or more primes, or a superscript. In the case of a two-letter abbreviation for a chemical compound, the abbreviation must be treated as if it were a letter.
- iv. The subscript must consist of numeric symbols only, and must carry no superscripts or subscripts of its own.

- (1) x_1 
 (x sub 1; subscript indicator not required because all conditions i-iv hold)

- (2) x_{11} 
 (x sub 1 1; subscript indicator not required because all conditions i-iv hold)
- (3) $\mathfrak{A}_1$ 
 (German capitalized ah sub 1; subscript indicator not required because all conditions i-iv hold)
- (4) x'_1 
 (x prime sub 1; subscript indicator not required because all conditions i-iv hold)
- (5) x''_2 
 (x double prime sub 2; subscript indicator not required because all conditions i-iv hold)
- (6) $_3x$ 
 (3 is a left subscript to x; subscript indicator is required because condition i does not hold)
- (7) x_{i1} 
 (x sub i sub 1; subscript indicator is required because condition ii does not hold)
- (8) $\log_2 x$ 
 (log to the base 2 of x; subscript indicator not required because all conditions i-iv hold)
- (9) 12_7 
 (12 sub 7; subscript indicator is required because condition iii does not hold)
- (10) $(\text{CO}_3)_2$ 
 (the carbonate radical taken twice; subscript indicator is required before the 2 because condition iii does not hold)
- (11) Na_2CO_3 
 (sodium carbonate; subscript indicator not required because all conditions i-iv hold)
- (12) seven_3 
 (seven sub 3; subscript indicator is required because condition iii does not hold)
- (13) x_{1j} 
 (x sub 1 sub j; subscript indicator is required because condition iv does not hold)
- (14) x_{2^n} 
 (x carries a subscript 2 which carries a superscript n; subscript indicator is required because condition iv does not hold)
- (15) $x_{2'}$ 
 (x sub 2 prime; subscript indicator is required because condition iv does not hold)

(16) $x_2 + k$ $\begin{smallmatrix} \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \\ \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \end{smallmatrix}$

(x carries a subscript of 2 plus k; subscript indicator is required because condition iv does not hold)

(17) $x_{1/2}$ $\begin{smallmatrix} \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \\ \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \end{smallmatrix}$

(x sub one-half; subscript indicator is required because condition iv does not hold)

(18) ${}_3x_1$ $\begin{smallmatrix} \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \\ \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \end{smallmatrix}$

(3 is a left subscript to x, 1 is a right subscript to x; subscript indicator is required before the 3 because condition i does not hold)

(19) A_{x1} $\begin{smallmatrix} \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \\ \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \end{smallmatrix}$

(A sub x 1; subscript indicator is required because condition iv does not hold)

(20) $x_{10,000}$ $\begin{smallmatrix} \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \\ \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \end{smallmatrix}$

(x sub 10,000; subscript indicator not required because all conditions i-iv hold)

(21) $x_{1.2}$ $\begin{smallmatrix} \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \\ \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \end{smallmatrix}$

(x sub 1.2; subscript indicator not required because all conditions i-iv hold)

(22) $x_{.6}$ $\begin{smallmatrix} \bullet\bullet & \bullet\bullet & \bullet\bullet \\ \bullet\bullet & \bullet\bullet & \bullet\bullet \end{smallmatrix}$

(x sub .6; subscript indicator not required because all conditions i-iv hold)

(23) $\sum_0^n a_k$ $\begin{smallmatrix} \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \\ \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \end{smallmatrix}$

(the summation from zero to n of a sub k; subscript indicator is not required because all conditions i-iv hold)

(24) $\prod_0^n a_k$ $\begin{smallmatrix} \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \\ \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \end{smallmatrix}$

(the product from zero to n of a sub k; subscript indicator is not required because all conditions i-iv hold)

(25) $3AF_{16}$ $\begin{smallmatrix} \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \\ \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \end{smallmatrix}$

(A and F represent a numeral in base 16; subscript indicator is required because condition iii does not hold)

(26) $\int_0^{\sqrt{1-x^2}} f(x) dx$ $\begin{smallmatrix} \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \\ \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \end{smallmatrix}$

(the integral from 0 to the square root of $1 - x^2$ of f of x dx; subscript indicator is required because condition iii does not hold)

§78. **Comma at Superscript or Subscript Level:** A commonly occurring superscript or subscript notation is the one in which two consecutive items are separated by a comma with an optional space following the comma. In this configuration, the symbol $\begin{smallmatrix} \bullet\bullet \\ \bullet\bullet \end{smallmatrix}$ (dots 2-4-6) must be used to replace the comma and the optional space used in this way. This contracted form must not be used to replace a comma and the optional space which follows it in a configuration which is on the base line.

(1) $x_{1, j, k}$ $\begin{smallmatrix} \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \\ \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet & \bullet\bullet \end{smallmatrix}$

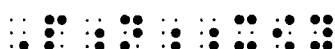
(each comma is followed by a space in ink print)

(2) $x_{(a,b)}$ 

(the comma is not followed by a space in ink print)

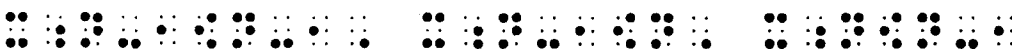
(3) $x_{1,2}$ 

(the comma is followed by a space in ink print)

(4) $P_{n_x,y}$ 

(the comma is followed by a space in ink print)

(5) $x_{n-1, n-1}, x_{n-1, n}, x_{n, n-1}$



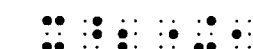
(the comma and space between the items on the base line cannot be contracted)

(6) (x, y) 

(the comma and space between the items on the base line cannot be contracted)

§79. Circumstances Determining Changes of Level: The symbols and situations listed below have the following effect in determining changes of level.

a. A level indicator terminates the effect of a previous level indicator and initiates the level implied by the new indicator. In the case of the base-line level, the previous base-line indicator may only be implied.

(1) $x^2 + 1$ 

(the superscript indicator terminates the previous implied base-line level and initiates the superscript level, the base-line indicator terminates the previous superscript level and initiates the base-line level)

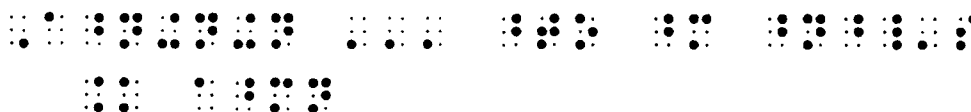
(2) $x_a + y^2$ 

(the subscript indicator terminates the previous implied base-line level and initiates the subscript level, the base-line indicator terminates the previous subscript level and initiates the base-line level, the superscript indicator terminates the previous base-line level and initiates the superscript level)

(3) $\frac{e^{x^2}}{2}$ 

(the superscript indicator terminates the previous implied base-line level and initiates the first-order superscript level, the second-order superscript indicator terminates the previous first-order superscript level and initiates the second-order superscript level, the base-line indicator terminates the previous second-order superscript level and initiates the base-line level)

(4) $A^{n+n+n \dots \text{to } m \text{ n's}} = a^{mn}$



(the superscript indicator which follows the n preserves the effect of the preceding superscript indicator; otherwise, the punctuation indicator would terminate the effect of the previous level indicator and initiate the base-line level)

d. The space which immediately follows a symbol of shape, an abbreviated function name, or an unabbreviated function name, provided the latter is in a mathematical context, preserves the level that is already in effect. If these items carry a superscript or subscript, the space which follows such a superscript or subscript reinstates the level that was in effect before.

(1) $b_{\triangle ABC}$

(the space preserves the subscript level at which the triangle appears)

(2) $\sin x$

(the space preserves the base-line level of sin)

(3) $\cos^2 x$

(the space reinstates the base-line level of cos)

(4) $e^{\sin x}$

(the space preserves the superscript level at which sin appears)

(5) $e^{\sin x} + i^{\cos x}$

(each space preserves the superscript level at which the abbreviated function names appear)

(6) $e^x + \ln x$

(the space preserves the superscript level at which ln appears)

(7) $e^{\cos^2 x}$

(the space reinstates the superscript level at which cos appears)

(8) $e^{\sin^2 x + \sin^2 y}$

(each space reinstates the superscript level at which the abbreviated function names appear)

(9) $q^{\log_q u}$

(the space reinstates the superscript level at which log appears)

(10) $V_{\max(m, n)}$

(the space preserves the subscript level at which max appears)

e. The space which occurs in a numeral for the purpose of dividing it into short regular segments preserves the level already in effect.

(1) $e^{3.14159\ 26535}$

f. The space which precedes an ellipsis or long dash preserves the effect of any previous level indicator. The space which follows the ellipsis or long dash preserves the level that is already in effect. However, if such a space is followed by literary text, unrelated mathematical text, or a sign of comparison, this space initiates the base-line level.

§80. Use of Level Indicators:

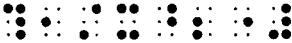
a. A level indicator must be used before any braille indicator or grouping symbol whenever this braille indicator or grouping symbol applies to a level other than the one currently in effect.

(1) $\sqrt{x^2 + y^2}$ 

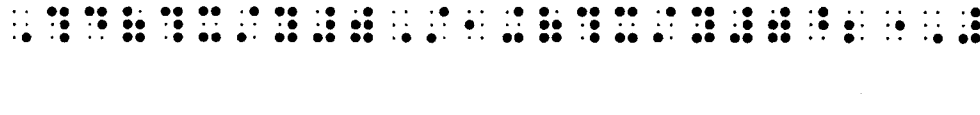
(the termination indicator applies to the base line, therefore the base-line indicator is required)

(2) $e^{\sqrt{x^2 + y^2}}$ 

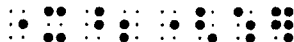
(the termination indicator applies to the first-order superscript level, therefore the superscript indicator is required)

(3) $\frac{1}{x^2}$ 

(the closing simple-fraction indicator applies to the base line, therefore the base-line indicator is required)

(4) $\frac{d\left(\frac{x}{y}\right)}{1 + \left(\frac{x}{y}\right)^2}$ 

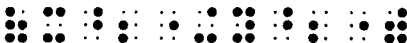
(the closing complex-fraction indicator applies to the base line, therefore the base-line indicator is required)

(5) $\overline{x^2}$ 

(the directly-over indicator applies to the base line, therefore the base-line indicator is required)

(6) $x^2 y^2$ 

(the opening and closing cancellation indicators apply to the base line, therefore the base-line indicator is required)

(7) $(x^2 + y^2)$ 

(the closing parenthesis applies to the base line, therefore the base-line indicator is required)

(8) $x^{(m^n)}$ 

(the closing parenthesis applies to the first-order superscript level, therefore the superscript indicator is required)

b. The superscript indicator must be used to restate the superscript level when two superscripts are consecutive but one applies to the expression which precedes it and the other applies to the expression which follows it. Similarly, the subscript indicator must be restated when two subscripts are consecutive and one applies to the expression preceding it and the other applies to the expression following it. A superscript or subscript indicator must be restated before a modified expression which is interior to the superscript or subscript expression, provided that the multipurpose indicator is also used.

(1) $p^b c^q$ 

(2) $P_b c^q$ 

(3) $P_{12} Q$ 

(4) A_{x+y} 

(the subscript indicator after the plus sign must be restated before the multipurpose indicator)

c. The appropriate level indicator must be used before each part of an abbreviation or phrase which is at a level other than the base line.

(1) $\triangle_{\text{regular polygon}}$

(2) $a^n + n + n \dots \text{to } m \text{ n's}$

d. Whenever spaces are left for the purpose of achieving alignment, level indicators must be used as though such spaces were not present.

(1)

$$\begin{array}{r}
 2x^3 - x^2 + x + 1 \\
 3x^3 + 4x^2 - 10x + 7 \\
 \quad 5x^2 \quad + 12 \\
 - 2x^3 \quad - 6x \\
 \hline
 3x^3 + 8x^2 - 15x + 20
 \end{array}$$

e. The appropriate level indicator must be used before any symbol or situation in which a change of level is required but the change is not effected by any of the conditions of §79.

(1) $360^\circ\text{-interval}$

(the base-line indicator places the hyphen at the base-line level)

(2) $\int_{u=a}$

(the subscript indicator before the equals symbol keeps this symbol at the subscript level; otherwise it would be at the base-line level)

(3) $t]_{t=a}^{t=b} = b - a$

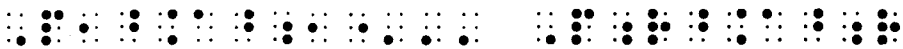
(the subscript and superscript indicators before the first two equals symbols keep these at the subscript and superscript levels respectively, while the space before the last equals symbol places it at the base-line level)

(4) $e^{\sin x} = a > y$

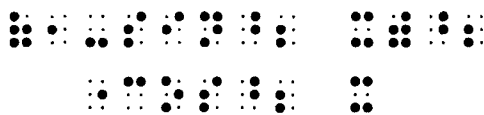
(the superscript indicator before the equals symbol keeps this symbol at the superscript level)

(5) $P_{s_1} \dots s_n$

(the subscript indicator before the ellipsis places the ellipsis at the first-order subscript level)

(6) $P_1^{\alpha_1} \dots P_r^{\alpha_r}$ 

(the base-line indicator places the ellipsis at the base-line level)

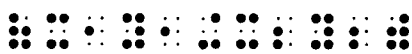
(7) $(1 - \sin^2 x)^2 \cos^2 x$ 

(it is assumed that this expression has had to be run over to another braille line at the place indicated; the base-line indicator at the beginning of the braille line places cos at the base-line level; otherwise it would have remained at the superscript level initiated on the previous braille line)

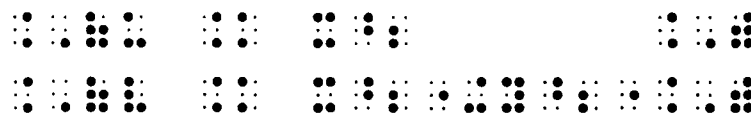
§81. Non-Use of Level Indicators:

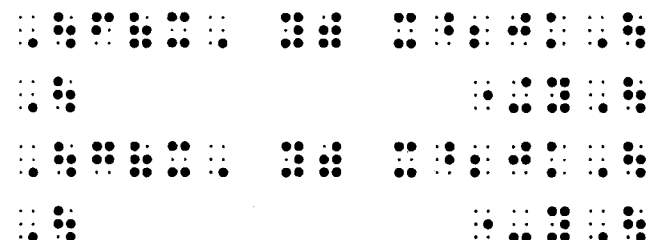
a. The base-line indicator must not be used to return to the base line from a numeric subscript if the subscript indicator has not been used before the numeric subscript.

(1) $(x_1 + 1)$ 
(base-line indicator not required before the plus symbol)

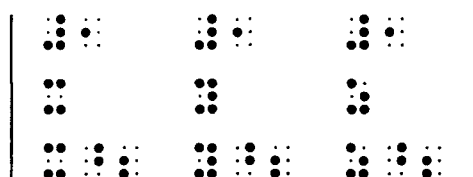
(2) $(x_1 y_1 + x_2 y_2)$ 
(base-line indicator not required after any of the numeric subscripts)

b. The base-line indicator must not be used before a right enlarged grouping symbol if this symbol either is separated from its preceding material by one or more spaces, or if the material which precedes the right grouping symbol is not the end of an expression.

(1) $\begin{cases} u = x^2 \\ v = x^2 + y^2 \end{cases}$ 

(2) $\begin{vmatrix} f(x, y) & x^{2jk} + y \\ g(x, y) & x^{2jk} - y \end{vmatrix}$ 

c. A level indicator must not be used before any closing grouping symbol which is drawn in.

(1) $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$ 

d. A level indicator must not be used to change the level if any symbol or situation specified in §79 has already effected the change to the desired level.

§82. Simultaneous and Non-Simultaneous Superscripts and Subscripts:

a. When an expression simultaneously carries a superscript and subscript, the subscript must be indicated first, even if the subscript is numeric and does not require the subscript indicator. However, if this sign carries one or more primes in addition, see §83.

(1) x_a^n $\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$

(x carries simultaneously a subscript of a and a superscript of n)

(2) ${}_a^n x$ $\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$

(x carries simultaneously a left subscript of a and a left superscript of n)

(3) x_1^2 $\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$

(x carries simultaneously a subscript of 1 and a superscript of 2)

b. When the same expression carries a superscript and a subscript which are not simultaneous, the relative horizontal positions of the signs must be retained in the transcription, but the base-line indicator must be inserted before making the transition to the other level.

(1) a_m^n $\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$

(the superscript is closer to the a than the subscript)

(2) a_m^n $\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$

(the subscript is closer to the a than the superscript)

(3) ${}_b^n x$ $\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$

(the left subscript is closer to the x than the left superscript)

(4) ${}_b^n x$ $\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$

(the left superscript is closer to the x than the left subscript)

(5) x_1^2 $\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$

(the subscript is closer to the x than the superscript)

(6) $x_a'^b$ $\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$

(the subscript is closer to x prime than the superscript)

§83. Primes in Addition to Superscripts or Subscripts:

a. The prime symbol must never be preceded by the superscript indicator.

(1) x' $\begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix}$

Modifiers

Arc

Concave upward



Concave downward



Arrow

Barbed at both ends



Barbed at left



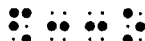
Barbed at left and dotted at right



Barbed at right



Contracted form



Uncontracted form



Dotted at both ends



Dotted at left (no barb)



Dotted at left and barbed at right



Dotted at right (no barb)



Hollow dot at both ends



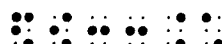
Hollow dot at left (no barb)



Hollow dot at left and barbed at right



Hollow dot at right and barbed at left



Hollow dot at right (no barb)



Bar

Horizontal (macron)



Vertical



Caret (circumflex)	^	⠠
Inverted	∨	⠡
Left-pointing	<	⠢
Right-pointing	>	⠣
Dot	•	⠠
Hollow Dot	◦	⠠
Question Mark	?	⠠
Tilde		
Extended	~	⠤
Simple	~	⠤
Triangle (equilateral)	△	⠤

§85. **Modifiers:** A modifier is a superscript or subscript which occupies, respectively, a position directly over or directly under the sign to which it applies. The modifiers in the list at the beginning of this rule are those most commonly used, but other modifiers must be treated in the same manner.

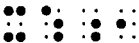
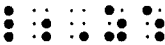
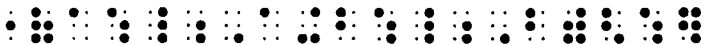
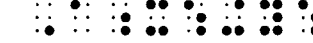
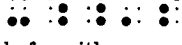
§86. **Modified Expressions:**

a. **The Five-Step Rule for Transcribing Modified Expressions:** The components of a modified expression must appear in the following order:

- i. Multipurpose indicator ⠠
- ii. Expression being modified.
- iii. Directly-over indicator ⠠ or directly-under indicator ⠡
- iv. Modifier.
- v. Termination indicator ⠠

These five components may never be separated from each other by transition to another braille line. The termination indicator terminates only the modified expression; it does not affect the level at which the modified expression occurs.

- (1) x ⠠ ⠠ ⠠ ⠠ ⠠ ⠠
 (x with subscribed bar)

- (7) $\bar{x}_1$ 
(x with superscribed bar, sub 1)
- (8) $\bar{x}_n$ 
(x with superscribed bar, sub n)
- (9) $\bar{Z}$ 
(boldface capital Z with superscribed bar)
- (10) $3.5\bar{4}$ 
(3.54, bar superscribed to the 4)
- (11) $(\bar{a}A + \bar{b}B)$ 
(a with superscribed bar times boldface capitalized A plus b with superscribed bar times boldface capitalized B, the whole expression with superscribed bar)
- (12) $A_{\bar{x}}$ 
(A with a right subscript of x with superscribed bar)
- (13) $A_{\bar{x} + \bar{y}}$ 
(A with a right subscript of x with superscribed bar plus y with superscribed bar)
- (14) $e^{a\bar{x}}$ 
(e with a right superscript of a times x with superscribed bar)
- (15) $\bar{x}'s$ 
(the plural of x with superscribed bar)

§87. Modifiers of Higher Order:

a. A modifier of the second order must be preceded by the second-order directly-over or directly-under indicator, and a modifier of the third order must be preceded by the third-order directly-over or directly-under indicator. The termination indicator, however, must be used only once, after the last modifier symbol.

- (1) $\frac{a=3}{x+y}$ 
(x plus y superscribed by a bar, which in turn is superscribed by a equals 3)
- (2) $\frac{x+y}{\frac{a=3}{b=2}}$ 
(x plus y subscribed by a bar, which in turn is subscribed by a equals 3, which in turn is subscribed by b equals 2)

b. A modifier of order higher than the third must be treated in the manner suggested in a above.

in the binomial coefficient; the expression which follows the directly-under indicator and precedes the closing parenthesis corresponds to the lower sign of the binomial coefficient.

$$(1) \binom{n}{k}$$

(the binomial coefficient with n as the upper sign and k as the lower sign)

$$(2) \binom{g_j}{a_j}$$

(the binomial coefficient with g sub j as the upper sign and a sub j as the lower sign)

§91. Modified Expressions in Superscripts and Subscripts: If a modified expression is part or all of a right superscript or subscript, the multipurpose indicator must be preceded by the appropriate level indicator. This will automatically be the case if the modified expression occurs at the beginning of the superscript or subscript; but the appropriate level indicator must be restated if the modified expression occurs at an interior position of the superscript or subscript. If the contracted form for a modified expression is used so that the multipurpose indicator does not appear, the appropriate level indicator must not be restated.

$$(1) A_{\tilde{x}}$$

(A carries a subscript of x with superscribed tilde)

$$(2) A_{\tilde{x}+\tilde{y}}$$

(A carries a subscript of x with superscribed tilde plus y with superscribed tilde; the subscript level after the plus sign must be restated before the multipurpose indicator)

$$(3) A_{\bar{x}+\bar{y}}$$

(A carries a subscript of x with superscribed bar plus y with superscribed bar)

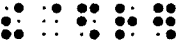
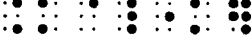
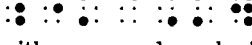
§92. Plural Modified Expressions: (See §39).

§93. Modification by Superposition: When one sign modifies another by superposition, in deciding which is the basic sign and which is the superposed sign, the following hierarchy, in descending order, should be used as a guide:

- i. Integral sign
- ii. Operation signs
- iii. Bars — horizontal and vertical
- iv. Shape signs
- v. Comparison signs
- vi. Signs not covered above

A sign belonging to a category lower on the list must be regarded as superposed on a sign higher on the list, and the superposition transcribed accordingly. If two signs belong to the same category, it is permissible to represent the superposition in either

order, provided that the same order is used consistently throughout the entire transcription. The components of a sign compounded by superposition must be joined by the superposition indicator and transcribed unspaced, and without transition to another braille line. The termination indicator must follow the second component. (For other examples, see "Comparison Signs Compounded by Superposition" pages 140-141, and 143.)

- (1)  
(integral sign with superposed rectangle)
- (2)  
(horizontal bar with superposed square)
- (3)  
(dot between bars of equals sign)
- (4)  
(equals sign with superposed inclusion sign)
- (5)  
(inclusion sign with superposed equals sign)
- (6)  
(angle with superposed arc)

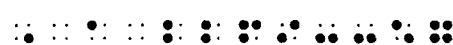
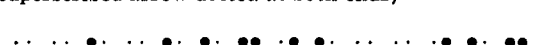
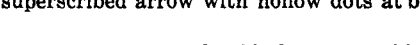
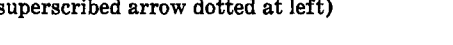
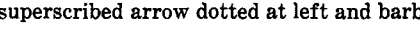
§94. Interior Modifiers with Signs of Shape: See §111.

§95. Arc:

- (1)  
(A with subscribed arc concave upward)
- (2)  
(AB with superscribed arc concave upward)
- (3)  
(A with subscribed arc concave downward)
- (4)  
(AB with subscribed arc concave downward)

§96. Arrows: Arrows must not be regarded as modifiers when they occur directly over or directly under a comparison sign. In that event, they become a component of a sign of comparison compounded vertically.

When a right-pointing arrow with a single shaft of ordinary length is in regular type, has a full barb, and is not part of a more complex construction or compound modifier, it must be transcribed in its contracted form. If such an arrow is in non-regular type, does not have a full barb or shaft of ordinary length, is part of a compound modifier, or is itself modified, it must be represented in its uncontracted form.

- (1) $\overrightarrow{AB}$ 
(AB with superscribed arrow barbed at right)
- (2) $\overleftarrow{AB}$ 
(AB with superscribed arrow barbed at left)
- (3) $\overleftrightarrow{AB}$ 
(AB with superscribed arrow barbed at both ends)
- (4) $\overleftarrow{\cdot}AB$ 
(AB with superscribed arrow barbed at the left and dotted at the right)
- (5) $\overleftarrow{\cdot}\overrightarrow{\cdot}AB$ 
(AB with superscribed arrow dotted at both ends)
- (6) $\overleftarrow{\circ}\overrightarrow{\circ}AB$ 
(AB with superscribed arrow with hollow dots at both ends)
- (7) $\overleftarrow{\cdot}AB$ 
(AB with superscribed arrow dotted at left)
- (8) $\overleftarrow{\cdot}\overrightarrow{AB}$ 
(AB with superscribed arrow dotted at left and barbed at right)
- (9) $\overrightarrow{\cdot}AB$ 
(AB with superscribed arrow dotted at right)
- (10) $X \overset{f}{\overset{\circ}{\rightarrow}} Y$ 
(arrow with superscribed f hollow dot g between X and Y)

§97. Horizontal Bar:

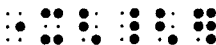
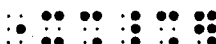
a. The horizontal bar must not be regarded as a modifier when it occurs directly over or directly under a comparison sign. In that event, it becomes a component of a sign of comparison compounded vertically (see §147). When the horizontal bar is itself modified by a dot under it or a caret directly over or under it, the combination is a modified sign of comparison (see §146). When the horizontal bar is itself modified by a dot over it, the combination is a sign of operation.

b. The horizontal bar is often used to indicate the recurrence of one or more digits in a decimal numeral by placing it over the digits which recur.

- (1) $\overline{.3}$ 
(decimal point 3, with a bar over the 3)
- (2) $\overline{.7128}$ 
(decimal point 7128, with a bar over the four digits)
- (3) 3.5729 
(3.5729, with a bar over the 29)

c. When the horizontal bar occurs over or under the integral sign, or over or under the abbreviated or unabbreviated function name for *limit*, the bar must not be treated as a modifier (see §171 and §118, respectively).

§98. Caret:

- (1) $\hat{x}$ 
(x with superscribed caret)
- (2) $\overset{\wedge}{=}$ 
(equals sign with superscribed caret)
- (3) $\overset{\vee}{x}$ 
(x with subscripts inverted caret)
- (4) $\overset{<}{=}$ 
(equals sign with superscribed left-pointing caret)
- (5) $\overset{>}{=}$ 
(equals sign with superscribed right-pointing caret)

§99. Dot:

a. The dot is frequently used to indicate the recurrence of one or more digits in a decimal numeral. When used for this purpose, a dot is usually placed in print over each digit of the recurring sequence. In braille, however, only a single dot must be used as a modifier.

- (1) $\dot{.3}$ 
(decimal point 3, with a dot over the 3)

- (2) $\dot{.135}$ 
 (decimal point 135, with a dot over each of the three digits)

- (3) $\dot{.135}$ 
 (decimal point 135, with a dot over the 5)

b. Although there is theoretically no limit to the number of dots which may be placed over or under a single mathematical expression, in practice the number rarely exceeds three dots. However, as many dots must be used in the transcription as are present in the printed text, except in the case of recurring decimals as in a.

- (1) $\ddot{x}$ 
 (x with two dots over it)

- (2) $\dddot{x}$ 
 (x with three dots over it)

- (3) $\underset{\cdot}{x}$ 
 (x with two dots under it)

§100. Hollow Dot:

- (1) $\overset{\circ}{=}$ 
 (equals sign with superscribed hollow dot)

§101. Question Mark:

- (1) $\overset{?}{=}$ 
 (equals sign with superscribed question mark)
- (2) $\underset{?}{=}$ 
 (equals sign with subscribed question mark)

§102. Tilde: The tilde, simple or extended, must not be regarded as a modifier when it occurs directly over or under a comparison sign. In that event, it becomes a component of a sign of comparison compounded vertically (see §147). When the tilde, simple or extended, is itself modified by a dot or a caret directly over or under it, the combination is a modified sign of comparison (see §146).

RULE XV—RADICALS

Radical (square root) $\sqrt{\quad}$ ⠠⠠⠠⠠⠠⠠

Radical Indicators

Index-of-Radical ⠠⠠⠠⠠⠠⠠

Order-of-Radical

First inner radical ⠠⠠⠠⠠⠠⠠

Second inner radical ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

Third inner radical ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

Termination ⠠⠠⠠⠠⠠⠠

§103. Simple Radicals: The most commonly occurring radical is the square root.

a. When the square root sign has a vinculum (horizontal bar) which specifies the extent to which the radical sign is effective, the transcription of such a radical is accomplished by the following three steps:

i. The radical symbol ⠠⠠⠠⠠⠠⠠

ii. The expression to which it applies (radicand).

iii. The termination indicator ⠠⠠⠠⠠⠠⠠

(1) $\sqrt{2}$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

(2) $\sqrt{x+y}$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

(3) $\sqrt{x^2+1}$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

(4) $\sqrt{x^2+y^2}$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

(5) $\sqrt{\frac{x}{y}}$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

(6) $3\sqrt{a}$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

(7) $\sqrt{x^3}$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

(3) $\sqrt{\sqrt[3]{x}} = \sqrt[3]{\sqrt{x}}$



(the square root of the cube root of x equals the cube root of the square root of x)

(4) $\sqrt{x + \sqrt{y + \sqrt{z}}}$



(a nest of three radicals; the outer radical contains x plus the inner radicals, the first inner radical contains y plus the second inner radical, and the second inner radical contains z)

RULE XVI—SHAPES

Shape Indicator



Interior Shape-Modification Indicator



Structural Shape-Modification Indicator



Filled-in Shape Indicator



Shaded Shape Indicator



Termination Indicator



Basic Shapes

Angle



Arc

Concave upward



Concave downward



Arrow

Left-pointing



Right-pointing

Contracted



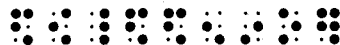
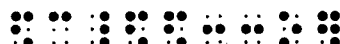
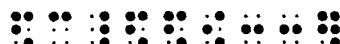
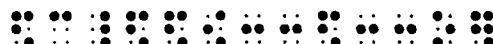
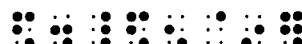
Uncontracted

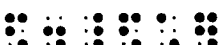
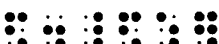
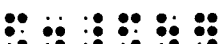
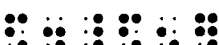


Down-pointing



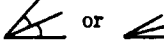
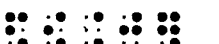
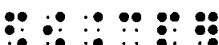
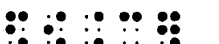
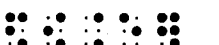
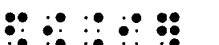
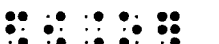
Up-pointing		
Circle		
Diamond		
Ellipse (oval)		
Hexagon		
Irregular		
Regular		
Intersecting Lines		
Is Parallel To		
Is Not Parallel To		
Is Perpendicular To		
Is Not Perpendicular To		
Parallelogram		
Pentagon		
Irregular		
Regular		
Quadrilateral		
Rectangle		
Rhombus	 or 	
Square		
Star		
Trapezoid		

Triangle**Inverted****Regular (equilateral)****Shapes with Interior Modification****Angle****Angle with interior arc****Angle with interior clockwise arrow****Angle with interior counterclockwise arrow****Circle****Circle with interior arrow pointing right****Circle with interior arrow pointing left****Circle with interior arrow pointing right over interior arrow pointing left****Circle with interior arrow pointing left over interior arrow pointing right****Circle with interior arrow pointing up****Circle with interior arrow pointing down****Circle with interior arrow pointing up followed by interior arrow pointing down****Circle with interior arrow pointing down followed by interior arrow pointing up****Circle with interior cross****Circle with interior dot****Circle with interior minus sign****Circle with interior plus sign****Square****Square with interior diagonals**

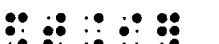
Square with interior dot		
Square with interior horizontal bar		
Square with interior vertical bar		
Square with interior northwest-southeast diagonal		
Square with interior southwest-northeast diagonal		

Shapes with Structural Modification

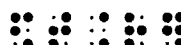
Angle

Adjacent angles	 or 	
Alternate exterior angles		
Alternate interior angles		
Complementary angles		
Corresponding angles		
Exterior angles		
Interior angles		
Obtuse angle		
Right angle		
Straight angle		
Supplementary angles		
Verticle angles		

Triangle

Acute triangle		
Isosceles triangle		
Obtuse triangle		

Right triangle



Scalene triangle



§106. **Basic Shapes:** A shape is a sign which is in general a miniature picture or diagram of the object which the sign represents.

A shape is represented by using a letter, numeral, or even a configuration of dots which is suggestive of the shape. The shape indicator must precede the shape symbol. A symbol of shape must be used only for the representation of the corresponding sign of shape; it must never be used to represent the word or phrase which is the name of such a sign of shape.

(1)  

(2)  

(3)  

(4)  

(5)  

(6)  

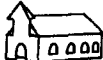
(7) $x \square$ 

(8) $\angle ABC = \angle DEF$        

§107. **Other Shapes:** Signs of shape which do not appear in the list of Basic Shapes must be represented by the use of one or more letters suggestive of the name of the shape being represented. Care must be exercised not to use an alphabetic symbol to which a meaning is already assigned in the above list. In addition, the transcriber must supply a note of explanation to the reader concerning the name of such a sign of shape and must supply a drawing of the shape if possible. If a combination of letters selected to represent a sign of shape is contractible, the contraction must not be used. The shape indicator must precede a shape symbol constructed in this way.

(1)  

(in ink print, the drawing of a moon)

(2)  

(in ink print, the drawing of a church)

§108. **Filled-In and Shaded Shapes:** Any of the closed shapes in the above list, if they are filled in or shaded, must be represented as such by using  (dots 4-5-6) or  (dots 4-6), respectively, preceding the shape symbol. The shape indicator must, in turn, precede whichever indicator has been used.

(1)  

(filled-in ellipse)

- (2)  
(shaded ellipse)

§109. **Polygons:** The list of Basic Shapes contains the shapes for regular polygons up to six sides. Any regular polygon with more than six sides must be represented in the manner suggested, that is, by using the numeral which specifies the number of sides. An irregular polygon, that is, one which has at least two unequal sides, two unequal angles, or both, must not be represented in this way. It must be represented as specified in §107.

- (1)  
(regular octagon)
- (2)  
(regular polygon of 12 sides)
- (3)  
(filled-in regular octagon)
- (4)  
(an irregular octagon; a transcriber's note with a drawing is required)

§110. **Shape with Structural Modification:** When a sign, which is a special case of a more general situation, is used, (for example, *right angle* is a special case of *angle*), or when two or more signs of shape are combined to form a composite sign with a more detailed structure, (for example, two *angles* are combined to form *adjacent angles*), the shape which is formed in either of these ways is called a *shape with structural modification*.

The modification is indicated by a letter or combination of letters suggestive of the nature of the modification. The symbol used for indicating the modification must be preceded by the structural shape-modification indicator and followed by the termination indicator. This combination must directly follow the symbol of basic shape which is being modified.

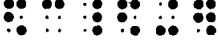
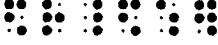
Shapes with structural modification not shown in the list of Shapes with Structural Modification must be transcribed in accordance with the principle suggested by those shape symbols. The transcriber must supply a note of explanation to the reader concerning the name of the structural modification and must supply a drawing if possible. If a combination of alphabetic symbols selected for a structural modifier constitutes a contractible combination, the contraction must not be used.

- (1)  
(an isosceles triangle; without modification the shape symbol signifies *triangle*)
- (2)  
(adjacent angles; without modification the shape symbol signifies *angle*)

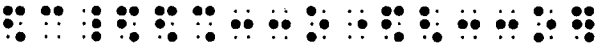
§111. **Shape with Interior Modification:**

- a. When a letter, operation sign, or other sign is placed inside the basic sign of shape, the shape which is formed in this way is called a *shape with interior modification*.

The modification is indicated by using the symbol which corresponds to the modifying sign. This symbol must be preceded by the interior shape-modification indicator and followed by the termination indicator. This combination must directly follow the symbol of basic shape which is being modified.

- (1)  
- (2)  
- (3)  
- (4)  

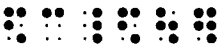
b. If two or more interior modifiers, arranged horizontally, occur inside the same basic sign of shape, the corresponding symbols must be separated by the multipurpose indicator, but the interior shape-modification indicator must be used only once, before the first modifying symbol. The entire combination must directly follow the basic symbol of shape which is being modified.

- (1)  

c. If two or more interior modifiers, arranged vertically, occur inside the same basic sign of shape, the corresponding symbols must be transcribed successively, without intervening spaces or indicators, beginning with the symbol which corresponds to the uppermost sign and proceeding in descending order. The first modifying symbol must be preceded by the interior shape-modification indicator, and the entire combination must directly follow the basic symbol of shape which is being modified. None of the interior signs may be regarded as a modifier of any of the others, and the technique for representing modified expressions does not apply.

- (1)  

§112. **Shape modified by Superposition:** When a sign is superposed upon a sign of shape, the shape which is formed in this way is called a *shape modified by superposition*. Superposition may be distinguished from interior modification by noting that in superposition one of the signs extends beyond the boundary of the other. Whereas, in the case of interior modification, one of the signs is confined within the boundary of the other (see §93).

- (1)  
(a vertical bar extending beyond the boundary of a circle)
- (2)  
(a circle with a vertical bar through the center extending as far as the boundary of the circle)

§113. **Drawn-In Shapes:** It is often better for the reader to have shapes drawn in than to have them represented by the elaborate braille constructions specified in this rule. However, it is not possible to formulate specific rules concerning which form should be used and, therefore, the decision is left to the experience and judgment of the transcriber.

§114. **Plural of a Sign of Shape:** The plural or the possessive of a sign of shape is sometimes indicated by placing the letter "s" on the inside of the sign of shape. When this form is employed, the braille transcription is effected simply by placing the lower-case letter "s" after the shape symbol (see §39).

- (1)  
(in ink print the "s" is inside the triangle shape)

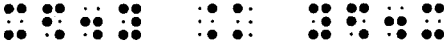
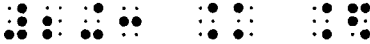
§115. Spacing with Symbols of Shape:

a. When a sign of shape is followed by its identification such as a letter, sequence of letters, or numeral, there must be a space between the shape symbol and its identification. In principle, the spacing rule which covers symbols of shape which are identified are the same as those which apply to function names and their abbreviations.

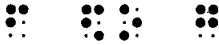
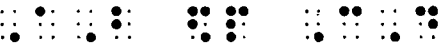
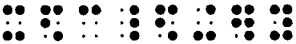
- (1) $\angle 1$
- (2) $\triangle ABC$
- (3) $\circ R$
- (4) In $\triangle ABC$, $\angle A = 90^\circ$.
- (5) $\triangle_{s} UVW$ and XYZ
- (6) $\perp A$
- (7) $\sphericalangle ABC$
- (8) $\angle x + \angle y$
- (9) $\angle 1 + 2\angle 3$
- (10) $\frac{\triangle ABC}{\triangle EFG}$
- (11) $m \angle ABC$
- (12) $\angle 90^\circ + \angle 120^\circ$

b. Shape symbols which represent omission must be spaced in accordance with the omitted item which they represent.

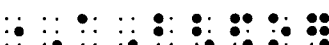
- (1) $\square\%$
(the square represents an omitted numeral)
- (2) $\$ \triangle$
(the triangle represents an omitted numeral)
- (3) $6 \frac{4}{12} = 6 \frac{\triangle}{3}$
(the triangle represents an omitted numeral)
- (4) $1 \text{ day} = 24 \diamond$
(the diamond represents an omitted word or abbreviation)

- (5) $x \square y = y \square x$ 
 (the square represents an omitted sign of operation)
- (6) $2 + 4 \triangle 7$ 
 (the triangle represents an omitted comparison sign)
- (7) $2 + 3 = \nabla$ 
 (the inverted triangle represents an omitted numeral)

c. Symbols of shape which are either comparison symbols or operation symbols must be spaced accordingly.

- (1) $f \rightarrow g$ 
 (the arrow is a comparison symbol)
- (2) $\lim_{x \rightarrow \infty} f(x)$ 
 (the arrow is a comparison symbol)
- (3) $AB \perp CD$ 
 ("is perpendicular to" is a comparison symbol)
- (4) $AB \nparallel CD$ 
 ("is not parallel to" is a comparison symbol)
- (5) $x \oplus y$ 
 (the circle with interior plus symbol is a symbol of operation)
- (6) $x \blacksquare y$ 
 (the filled-in square symbol is a symbol of operation)

d. In any case, a symbol of shape must be unspaced from any braille indicator which applies to it.

- (1) $\overrightarrow{AB}$ 
- (2) $1101_{\diamond} + 1000_{\diamond}$ 

RULE XVII—FUNCTION NAMES AND THEIR ABBREVIATIONS

Abbreviation	Function Name	Braille Equivalent
amp	amplitude	
antilog	antilogarithm	

Abbreviation	Function Name	Braille Equivalent
arc	arc	⠠⠠⠠⠠⠠⠠
arg	argument	⠠⠠⠠⠠⠠⠠
colog	cologarithm	⠠⠠⠠⠠⠠⠠⠠⠠
cos	cosine	⠠⠠⠠⠠⠠
cosh	hyperbolic cosine	⠠⠠⠠⠠⠠⠠⠠
cot	cotangent	⠠⠠⠠⠠⠠
coth	hyperbolic cotangent	⠠⠠⠠⠠⠠⠠⠠
covers	coversine	⠠⠠⠠⠠⠠⠠⠠⠠
csc	cosecant	⠠⠠⠠⠠⠠
csch	hyperbolic cosecant	⠠⠠⠠⠠⠠⠠⠠
ctn	cotangent	⠠⠠⠠⠠⠠
ctnh	hyperbolic cotangent	⠠⠠⠠⠠⠠⠠⠠
det	determinant	⠠⠠⠠⠠⠠
erf	error function	⠠⠠⠠⠠⠠
exp	exponential	⠠⠠⠠⠠⠠
exsec	exsecant	⠠⠠⠠⠠⠠⠠⠠
grad	gradient	⠠⠠⠠⠠⠠⠠
hav	haversine	⠠⠠⠠⠠⠠
im	imaginary part	⠠⠠⠠⠠
inf	infimum	⠠⠠⠠⠠⠠
lim	limit	⠠⠠⠠⠠⠠
$\overline{\text{lim}}$	upper limit	⠠⠠⠠⠠⠠⠠⠠

Abbreviation	Function Name	Braille Equivalent
<u>lim</u>	lower limit	⠠⠇⠠⠇⠠⠇
ln	natural logarithm	⠠⠇⠠⠇
log	logarithm	⠠⠇⠠⠇
max	maximum	⠠⠇⠠⠇
min	minimum	⠠⠇⠠⠇
mod	modulo	⠠⠇⠠⠇
re	real part	⠠⠇⠠⠇
sec	secant	⠠⠇⠠⠇
sech	hyperbolic secant	⠠⠇⠠⠇⠠⠇
sin	sine	⠠⠇⠠⠇
sinh	hyperbolic sine	⠠⠇⠠⠇⠠⠇
sup	supremum	⠠⠇⠠⠇
tan	tangent	⠠⠇⠠⠇
tanh	hyperbolic tangent	⠠⠇⠠⠇⠠⠇
vers	versine	⠠⠇⠠⠇

§116. Contractions in Function Names and Their Abbreviations: See §55b and §56.

§117. Numeric Subscripts with Function Names and Their Abbreviations: See §77.

§118. Modifiers with Function Names and Their Abbreviations: The bar which occurs over or under the function name "limit" or its abbreviation "lim" must not be treated as a modifier; the combination must be transcribed by means of special symbols for *upper limit* ⠠⠇⠠⠇ or ⠠⠇⠠⠇ or *lower limit* ⠠⠇⠠⠇ or ⠠⠇⠠⠇. Other modifiers, however, must be transcribed in accordance with the techniques for the representation of modified expressions.

- (1) $\overline{\lim}_{n \rightarrow \infty} f_n(x)$
- (2) $\lim_{n \rightarrow \infty} f_n(x)$
- (3) $\lim_{x \rightarrow 0} f(x)$

§119. Spacing with Function Names and Their Abbreviations:

a. A space must be left after an unmodified function name or its abbreviation. If the function name or its abbreviation carries a superscript, subscript, or modifier, the space must follow the superscript, subscript, or termination of modifier.

- (1) $\sin x$
- (2) $\cos^2 x$
- (3) $e^{\sin x}$
- (4) $\arcsin x$
- (5) $\log_a x$
- (6) $\lim_{x \rightarrow 0} f(x)$

b. If two or more consecutive function names or their abbreviations occur, the space between them may either be omitted or included in accordance with the ink print copy. When there is doubt concerning the presence of a space in ink print between the function names or their abbreviations, a space should be left in the transcription.

- (1) $\arcsin x$
(no space in ink print between arc and sin)
- (2) $\arcsin x$
(space between arc and sin clearly shown in ink print)

c. The expression which follows or precedes the function name or its abbreviation must be spaced in accordance with the other spacing rules of this Code.

- (1) $\sin x + y$
(in ink print, there is a space on both sides of the plus sign)
- (2) $\sin \pi/3$
(in ink print, there is no space on either side of the diagonal line)

(3) $\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$

(in ink print, $\cos 45^\circ$ and $\cos 30^\circ$ are preceded and followed by spaces)

(4) $2\sin x + 3\cos y$

(in ink print, there is no space after the 2 and the 3, and there is a space on both sides of the plus sign)

(5) $\sin a \cos y$

(in ink print, there is a space on both sides of the a and the y)

(6) $\sqrt{1 - \cos^2 x}$

(in ink print, there is a space on both sides of the minus sign and the x)

(7) $\frac{1}{\cos} - \cos = \tan \cdot \sin$

(8) $1/\cos - \cos = \tan \cdot \sin$

RULE XVIII—SIGNS AND SYMBOLS OF GROUPING

		Normal	Enlarged
Parentheses (round brackets)			
Left	(		
Right	)		
Brackets (square brackets)			
Left	[		
Right	]		
Boldface Left	[		
Boldface Right	]		

		Normal	Enlarged
Braces (curly brackets)			
Left	}		
Right	{		
Vertical Bar			
Single			
Double			
Boldface Single			
Boldface Double			
Angle Brackets (angular parentheses)			
Left	<		
Right	>		
Barred Brackets			
Left			
Right			
Barred Braces			
Left			
Right			
Half Brackets			
Upper Left	┌ or ⌈		
Upper Right	┐ or ⌋		
Lower Left	└ or ⌞		
Lower Right	┘ or ⌟		
Transcriber's Grouping Symbols			
Left			
Right			

(2) A_{n11}

(3) $[a_1 a_2 \dots a_n]$

§124. Vertical Bars:

- a. Double boldface vertical bars are usually read as *the norm of*.

(1) $\| f \|$

- b. Single vertical bars are often read as *the absolute value of*, but are used for other purposes.

(1) $|x|$

(2) $\begin{vmatrix} 1 \\ 0 \end{vmatrix}$

(3) $\left|_{x=0} \right.$

§125. **Transcriber's Grouping Symbols:** The regular transcriber's grouping symbols in the above list must be used to enclose any transcriber's note which has been inserted into the text (see §186b). These must not be used to enclose a list of transcriber's notes which appears at the beginning of a braille volume. The same rules which govern punctuation and contraction of expressions containing grouping symbols also govern transcriber's notes. For use of enlarged transcriber's grouping symbols see §184b.

§126. **Use of Enlarged Grouping Symbols:** When a system of mathematical expressions is arranged on two or more lines of ink print, and a sign of grouping is used to unify the system, the corresponding grouping symbol in the transcription must be indicated as enlarged by the use of dot 6 to indicate the enlargement. Among such systems of mathematical expressions are: systems of equations, determinants, and matrices. Each braille line which contains any part of the transcription of such a system must also contain the enlarged grouping symbol and these must be vertically aligned. If only the left or only the right member of a pair of grouping signs is present in ink print, only the corresponding grouping symbol must be represented in the transcription. However, when it is advisable for any reason to do so, for example, to save space by avoiding runovers, the enlarged grouping symbols may be drawn.

(1) $\begin{cases} x + y = 2 \\ x - y = 0 \end{cases}$

(a two-line system of equations enclosed within braces, equations are accidentally aligned; in ink print, period is centered)

(2) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

(a two-by-two determinant enclosed within vertical bars; the equals sign and the $ad - bc$ are centered in ink print)

(3) $y = \begin{cases} x, & \text{if } x \leq 0 \\ 0, & \text{if } x > 0. \end{cases}$

(a two-line system unified on the left by a left bracket)

$$(4) \quad x = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a three-by-three matrix enclosed within large brackets)

$$(5) \quad \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = I.$$

(a three-by-three matrix enclosed within double vertical bars)

§127. Non-Use of Enlarged Grouping Symbols: Signs of grouping must not be indicated as enlarged in the transcription when the corresponding signs in ink print are made large such as for the purpose of covering a fraction, binomial coefficient or other material occupying a large amount of vertical space. No signs, except grouping signs, may ever be shown as being enlarged.

$$(1) \quad \left(\frac{x+y}{u+v} \right)^2$$

$$(2) \quad \binom{n+k}{k}$$

$$(3) \quad \sqrt{\sqrt{x}}$$

$$(4) \quad \int \frac{f(x) dx}{(x-t)^n}$$

§128. Spacing with Symbols of Grouping:

a. Spaces may be required to be left after an opening enlarged grouping symbol or before a closing enlarged grouping symbol to preserve the vertical alignment of such symbols required in §126.

$$(1) \quad \begin{cases} 2x + y = 9 \\ x - 3y = 11 \end{cases}$$

(a unified system of two equations in which vertical alignment is required)

b. A space must be left between an opening and closing grouping symbol when there is a blank, not representing omission, between the corresponding signs in ink print.

(1) { } ⠠⠠⠠⠠ ⠠⠠⠠⠠
(the empty set)

RULE XIX—SIGNS AND SYMBOLS OF OPERATION

Ampersand (and, logical product)	&	⠠⠠⠠⠠
Asterisk	*	⠠⠠⠠⠠
Back Slash (divides, is a factor of)	\	⠠⠠⠠⠠
Circle with Interior Dot	⊙	⠠⠠⠠⠠⠠⠠⠠⠠⠠
Circle with Interior Plus	⊕	⠠⠠⠠⠠⠠⠠⠠⠠⠠
Circle with Interior Minus	⊖	⠠⠠⠠⠠⠠⠠⠠⠠⠠
Dagger		
Single	†	⠠⠠⠠⠠
Double	‡	⠠⠠⠠⠠⠠⠠
Division (divided by)	÷	⠠⠠⠠⠠
Dot (and)	•	⠠⠠
Fraction Line (over)		
Diagonal	/	⠠⠠⠠⠠
Simple	—	⠠⠠
Diagonal Complex	/	⠠⠠⠠⠠⠠⠠
Complex	—	⠠⠠⠠⠠
Hypercomplex	—	⠠⠠⠠⠠⠠⠠
Hollow Dot	°	⠠⠠⠠⠠

Intersection (cap)	$\cap$	
Logical Product (and, meet)	$\wedge$	
Logical Sum (join, or)	$\vee$	
Minus		
Regular	$-$	
Boldface	$-$	
Minus Followed by Plus		
Boldface Minus Followed by Boldface Plus	$- +$	
Boldface Minus Followed by Regular Plus	$- +$	
Regular Minus Followed by Regular Plus	$- +$	
Regular Minus Followed by Boldface Plus	$- +$	
Minus or Plus	$\mp$	
Minus with Dot over (proper difference)	$\dot{-}$	
Multiplication (times)		
Cross (cartesian product)	$\times$	
Dot	$\cdot$	
Number Sign; Crosshatch; Tic-tac-toe; Pounds (weight)	$\#$	
Paragraph Mark	$\P$	
Plus		
Regular	$+$	
Boldface	$+$	
Plus Followed by Minus		
Boldface Plus Followed by Boldface Minus	$+ -$	
Boldface Plus Followed by Regular Minus	$+ -$	

(4) $f \circ g$ 

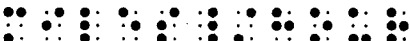
(5) $\sin x - \sin y$ 

(in ink print, there is a space on both sides of the minus sign)

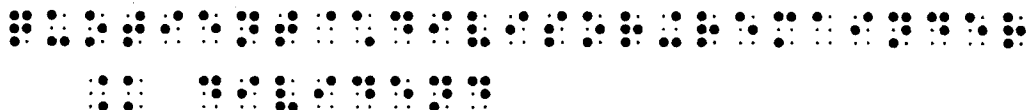
(6) $x \square y$ 

(7) $\square + \triangle$ 

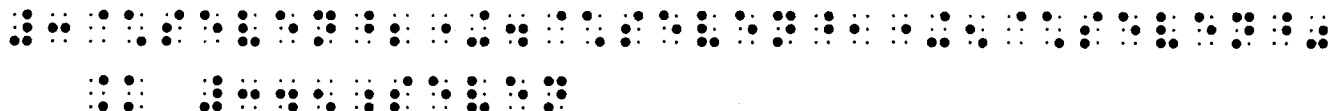
(8) $\text{rate} \times \text{time}$ 

(9) miles/hour 

(10) $\text{quotient} \times \text{divisor} + \text{remainder} = \text{dividend}$



(11) $3 \times \text{seven}^2 + 4 \times \text{seven}^1 + 5 \times \text{seven}^0 = 345_{\text{seven}}$



(12) $(2n + 3) \mid 3$ 

(13) $3 \text{ ft}^2 + 3 \text{ ft}^2 = 6 \text{ ft}^2$



RULE XX—SIGNS AND SYMBOLS OF COMPARISON

Simple Comparison Signs

Arc

Concave upward



Concave downward



Arrow

Left-pointing



Right-pointing		
Contracted	→	
Uncontracted	→	
Down-pointing	↓	
Up-pointing	↑	
Two-way		
Horizontal	↔	
Vertical	↕	
Equals (is equal to)		
Normal	=	
Boldface	≡	
Greater Than (is greater than)		
Normal	>	
With curved sides	⋈	
Identity (is congruent to; is identical to)	≡	
Inclusion (is contained in; is a subset of)	⊂	
Less Than (is less than)		
Normal	<	
With curved sides	⋈	
Membership (is an element of; belongs to)	ε or E or ∈	
Parallel To (is parallel to)	∥	
Perpendicular To (is perpendicular to)	⊥	
Proportion (as)	∴	
Ratio (is to)	:	

Relation (is related to)	R	
Reverse Inclusion (contains; in logic, implies)	$\supset$	
Reverse Membership (contains the element)	$\ni$ or $\ni$ or $\ni$	
Tilde		
Simple (is related to; is similar to)	$\sim$	
Extended (is related to)	$\approx$	
Variation (varies as)	$\propto$	
Vertical Bar (such that)		

Modified Comparison Signs**Equals Sign**

Caret over	$\hat{=}$	
Caret under (is projective to, projective correspondence)	$\hat{=}$	
Degree sign over (is equal in degrees to)	$\overset{\circ}{=}$	
Dot over (is approximately equal to)	$\overset{\cdot}{=}$	
Dot over and dot under	$\overset{\cdot}{\underset{\cdot}{=}}$	
Equilateral triangle over	$\triangle$	
Inverted caret over	∇	
Left-pointing caret over	$\lessgtr$	
Question mark over	$\overset{?}{=}$	
Right-pointing caret over	$\gtrless$	
Two dots over and two dots under	$\ddot{=}$	
Vertical bar over	$\overline{=}$	

Horizontal Bar

Caret over	$\hat{=}$	
------------	-----------	--

Caret under (is perspective to,
perspective correspondence)



Dot under



Simple Tilde, Dot Under



Comparison Signs Compounded Vertically

Arrow Combinations

Right-pointing over left-pointing



Right-pointing with upper barb over
left-pointing with lower barb



Right-pointing over boldface left-pointing



Left-pointing over boldface right-pointing



Boldface right-pointing over left-pointing



Boldface left-pointing over right-pointing



Boldface right-pointing over boldface left-pointing



Boldface left-pointing over boldface right-pointing



Long right-pointing over short left-pointing



Short right-pointing over long left-pointing



Equivalence (is equivalent to)



Greater Than

Bar over greater than (is equal to or greater than) $\bar{>}$ or $\bar{\geq}$



Bar under greater than (is
greater than or equal to) $\geq$ or $\bar{\geq}$



Equals sign over greater than
(is equal to or greater than) $\bar{=}$ or $\bar{\geq}$



Equals sign under greater than
(is greater than or equal to) $\geq$ or $\bar{=}$



Inclusion

Bar over inclusion (is a subset of)

 $\supset$ 

Bar under inclusion (is a subset of)

 $\supset$ 

Equals sign over inclusion (is a subset of)

 $\supset$ 

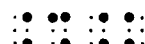
Equals sign under inclusion (is a subset of)

 $\supset$ **Intersection (cap)**

Bar under intersection

 $\cap$ 

Equals sign under intersection

 $\cap$ **Less Than**

Bar over less than (is equal to or less than)

 $\leq$ or $\leq$ 

Bar under less than (is less than or equal to)

 $\leq$ or $\leq$ Equals sign over less than
(is equal to or less than) $\leq$ or $\leq$ Equals sign under less than
(is less than or equal to) $\leq$ or $\leq$ **Logical Product (meet)**

Bar over logical product

 $\wedge$ 

Bar over and bar under logical product

 $\wedge$ 

Bar over and equals sign under logical product

 $\wedge$ 

Bar under logical product

 $\wedge$ 

Equals sign over logical product

 $\wedge$ 

Equals sign over and bar under logical product

 $\wedge$ 

Equals sign over and equals sign under logical product

 $\wedge$ 

Equals sign under logical product

 $\wedge$ 

Logical Sum (join)

Bar over logical sum	$\overline{\vee}$	
Bar over and bar under logical sum	$\overline{\vee}$	
Bar over and equals sign under logical sum	$\overline{=}$	
Bar under logical sum	$\underline{\vee}$	
Equals sign over logical sum	$=\vee$	
Equals sign over and bar under logical sum	$=\underline{\vee}$	
Equals sign over and equals sign under logical sum	$=\underline{=}$	
Equals sign under logical sum	$\underline{=}$	

Reverse Inclusion

Bar over reverse inclusion	$\supset$	
Bar under reverse inclusion	$\supset$	
Equals sign over reverse inclusion	$\supset$	
Equals sign under reverse inclusion	$\supset$	

Tilde (is related to)

Bar over double tilde	$\approx$	
Bar over single tilde	$\approx$	
Bar under double tilde	$\approx$	
Bar under single tilde	$\approx$	
Double tilde	$\approx$	
Equals sign over double tilde	$\approx$	
Equals sign over single tilde	$\approx$	
Equals sign under double tilde	$\approx$	
Equals sign under single tilde	$\approx$	

Union (cup)**Bar under union**U**Equals sign under union**=**Comparison Signs Compounded Horizontally****Arrow Combinations****Up-pointing followed by down-pointing**

↑ ↓

**Down-pointing followed by up-pointing**

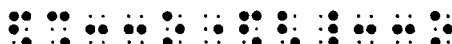
↓ ↑

**Up-pointing followed by boldface down-pointing**

↑ ↓

**Down-pointing followed by boldface up-pointing**

↓ ↑

**Boldface up-pointing followed by down-pointing**

↑ ↓

**Boldface down-pointing followed by up-pointing**

↓ ↑

**Boldface up-pointing followed by boldface down-pointing**

↑ ↓

**Boldface down-pointing followed by boldface up-pointing**

↓ ↑

**Greater Than****Followed by less than**

> <

**Followed by equals sign followed by less than**

> = <

**Less Than****Followed by greater than**

< >

**Followed by equals sign followed by greater than**

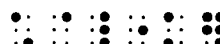
< = >

**Comparison Signs Compounded by Superposition****Dot****Between bars of equals sign**

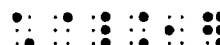
=

**Within inclusion sign**

⊆

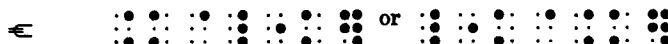
**Within reverse inclusion sign**

⊇

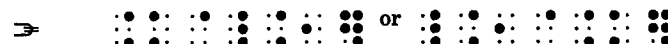


Equals Sign

Through inclusion sign



Through reverse inclusion sign

**Greater Than**Nest of two with straight sides
(is large compared with)

Nest of two with curved sides

**Horizontal Bar**

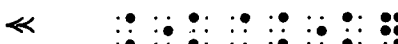
Through inclusion sign



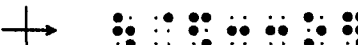
Through reverse inclusion sign

**Less Than**Nest of two with straight sides
(is small compared with)

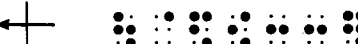
Nest of two with curved sides

**Vertical Bar**

Through shaft of right-pointing arrow



Through shaft of left-pointing arrow



§139. Negation: Comparison signs may be negated by a vertical stroke or by an oblique stroke in either direction. However the negation is effected in ink print, the symbol $\cdot\cdot$ (dots 3-4) must be placed unspaced before the comparison symbol being negated.

(1) $\neq$ $\cdot\cdot$

(oblique negation sign in ink print, from lower left to upper right)

(2) $\neq$ $\cdot\cdot$

(vertical negation sign in ink print)

(3) $\neq$ $\cdot\cdot$

(oblique negation sign in ink print, from upper left to lower right)

(4) $\neq$ $\cdot\cdot$

(oblique negation sign in ink print, from lower left to upper right)

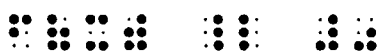
- (5)  
 (oblique negation sign in ink print, from upper left to lower right)
- (6)  
 (vertical negation sign in ink print)

§140. **Arrows:** A detailed discussion of the construction of arrows of many types is presented in Rule XXI. The arrows in the list of simple comparison signs are those which occur with the greatest frequency.

If a right-pointing arrow has a full barb and a single shaft of ordinary length, is in regular type, and occurs by itself, it must be represented in its contracted form. If such an arrow is in nonregular type, is itself modified, or occurs as part of a more complex modification, it must be represented in its uncontracted form.

- (1) $B \leftarrow A$ 
- (2) $A \rightarrow B$ 
- (3) $A \leftrightarrow B$ 
- (4) $X \xrightarrow{f \circ g} Y$ 

§141. **Identity:** This sign must not be used for *is congruent to* in geometry if another sign is employed for this purpose in ink print.

- (1) $f(x) \equiv 0$ 
- (2) $2 \equiv 5 \pmod{3}$ 

§142. **Membership:** This sign must not be mistaken for the Greek lower-case epsilon, even though it is sometimes referred to by that very name. This sign is generally used when speaking about sets and the elements of which they are composed. When the Greek lower-case epsilon is used in the same textbook, the publisher usually makes a sufficient distinction between the two signs to prevent this confusion.

- (1) $x \in A$ 
 (x is an element of A)

§143. **Relation:** When a letter or other sign is used between two expressions to show that they are related, the letter or sign used in this way must be regarded as a comparison sign. As such, it is subject to all the rules governing comparison signs and symbols. The letter R is frequently used in this situation.

- (1) $a R b$ 
- (2) $a \theta b$ 

(1) $x \sim y$   

(2) $x \rightsquigarrow y$   

(1) $\{x \mid |x| < 10\}$  (2) Choose $x \mid x = y^2$.  (3) $\exists x \mid x = -x$ 

§151. Spacing with Symbols of Comparison: A space must be left on either side of a comparison symbol. However, a space must not be left between the comparison symbol and any punctuation symbol, grouping symbol, or indicator which applies to it.

RULE XXI—ARROWS

Arrow Components

Arrow Direction Indicators

Depresses nearer arrowhead by 45 degrees



Elevates nearer arrowhead by 45 degrees



Makes nearer arrowhead point up



Makes nearer arrowhead point down



Arrow Shafts

Curved

(or)



Dashed

--



Dotted

...



Long double

==



Long single

—



Ordinary double

==



Ordinary single

—



Short double

==



Short single

—



Wavy

~



Arrow Types

Boldface



Regular (no indicator)

Arrowheads

Barbed left full

<



Barbed left lower

↙



Barbed left upper	⌞	⠠⠠⠠⠠
Barbed right full	⌟	⠠⠠
Barbed right lower	⌞	⠠⠠⠠⠠
Barbed right upper	⌟	⠠⠠⠠⠠
Blunted left full	[	⠠⠠
Blunted left lower	L	⠠⠠⠠⠠
Blunted left upper	[	⠠⠠⠠⠠
Blunted right full	]	⠠⠠
Blunted right lower	L	⠠⠠⠠⠠
Blunted right upper	L	⠠⠠⠠⠠
Curved left full	(	⠠⠠
Curved left lower	(	⠠⠠⠠⠠
Curved left upper	(	⠠⠠⠠⠠
Curved right full	)	⠠⠠
Curved right lower	)	⠠⠠⠠⠠
Curved right upper	)	⠠⠠⠠⠠
Straight left full		⠠⠠
Straight left lower		⠠⠠⠠⠠
Straight left upper		⠠⠠⠠⠠
Straight right full		⠠⠠
Straight right lower		⠠⠠⠠⠠
Straight right upper		⠠⠠⠠⠠

§152. **Contracted Form of Right-Pointing Arrow:** When a right-pointing arrow in regular type, with a single shaft of ordinary length and a full barb, occurs by itself, it must be represented in its contracted form . If such an arrow is in non-regular type, is modified, or occurs as part of a more complex modification, it must be represented in its uncontracted form .

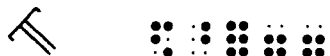
§153. **Arrow Components:** There is a large class of signs in the form of arrows which differ from each other in several ways. In the above list, there is presented an assortment of arrow components from which such signs are constructed. The entire construction is a simple comparison symbol.

§154. **Six Steps for Construction of Arrows:** Arrow components must be transcribed in the following order:

- i. The shape indicator.
- ii. The arrow direction, if it must be indicated.
- iii. The arrow type, if it must be indicated.
- iv. The left arrowhead, if any.
- v. The arrow shaft, if required.
- vi. The right arrowhead, if any.

(1) 
(arrow, two-way vertical, boldface, barbed arrowheads at both ends)

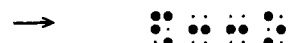
(2) 
(arrow, two-way horizontal, regular type, curved arrowheads at both ends)

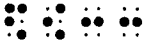
(3) 
(spear, northwest, blunted arrowhead)

§155. **Arrow Directions:** It is possible to represent eight arrow directions by making proper use of the direction indicators.

- a. The two horizontal directions, *right* and *left*, require no indicator.

(1) 
(right-pointing arrow, contracted form)

(2) 
(right-pointing arrow, uncontracted form)

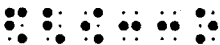
(3)  
 (left-pointing arrow)

(4)  
 (two-way horizontal arrow)

b. The two vertical directions, *up* and *down*, require the directly-over indicator or the directly-under indicator, respectively. The directly-over indicator "makes the arrowhead point up"; the directly-under indicator "makes the arrowhead point down". If a vertical arrow is printed with one arrowhead, it must be transcribed by using the appropriate symbol for a *right* arrowhead, and not a left one.

(1)  
 (arrow pointing up)

(2)  
 (arrow pointing down)

(3)  
 (vertical two-way arrow)

c. The four oblique directions require the superscript indicator or the subscript indicator. The superscript indicator "elevates the nearer arrowhead (if there are two) by 45 degrees from the horizontal position"; the subscript indicator "depresses the nearer arrowhead (if there are two) by 45 degrees from the horizontal position".

(1)  
 (arrow, northeast)

(2)  
 (arrow, northwest)

(3)  
 (arrow, southeast)

(4)  
 (arrow, southwest)

(5)  
 (arrow, northwest-southeast)

(6)  
 (arrow, southwest-northeast)

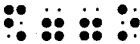
§156. Arrow Shafts: An arrow shaft may be curved, dashed, dotted, straight or wavy, single or double, long or short.

a. If an arrow shaft is curved, the direction of curvature is indicated by a left arrowhead or a right arrowhead. A curved arrow shaft followed by a right arrowhead represents a counterclockwise arrow; a curved arrow shaft preceded by a left arrowhead represents a clockwise arrow.

(1)  
(arrow, counterclockwise)

(2)  
(arrow, clockwise)

b. Most arrow shafts are single. An arrow with a double arrow shaft is sometimes called a *spear*.

(1)  
(spear, right-pointing)

(2)  
(spear, left-pointing)

(3)  
(spear, horizontal two-way)

c. Where the length of an arrow shaft has significance, the length is indicated by the number of repetitions of the braille arrow shaft symbol. The list distinguishes three lengths, but other lengths may be indicated by repeating the braille arrow-shaft symbol a suitable number of times.

(1)  
(short arrow, right-pointing)

(2)  
(short arrow, left-pointing)

(3)  
(short arrow, horizontal two-way)

(4)  
(long arrow, right-pointing)

(5)  
(long arrow, left-pointing)

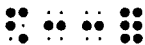
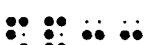
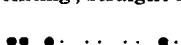
(6)  
(long arrow, horizontal two-way)

§157. **Arrow Types:** Most arrows are printed in regular type. In that case, no indicator is required. If an arrow is printed in boldface type, the boldface type indicator is required.

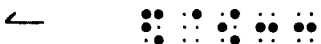
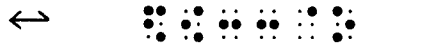
- (1)  
(boldface arrow, right-pointing)
- (2)  
(boldface arrow, left-pointing)
- (3)  
(boldface arrow, horizontal two-way)

§158. **Arrowheads:**

a. Most arrowheads are barbed. However, arrowheads also occur as blunted, curved, or straight. They may occur at the left end, right end, or at both ends, of the arrow shaft.

- (1)  
(arrow, right-pointing; blunted arrowhead)
- (2)  
(arrow, left-pointing; blunted arrowhead)
- (3)  
(arrow, horizontal two-way; blunted arrowheads)
- (4)  
(arrow, right-pointing; curved arrowhead)
- (5)  
(arrow, left-pointing; curved arrowhead)
- (6)  
(arrow, horizontal two-way; curved arrowheads)
- (7)  
(arrow, right-pointing; straight arrowhead)
- (8)  
(arrow, left-pointing; straight arrowhead)
- (9)  
(arrow, horizontal two-way, straight arrowheads)

b. An arrowhead with its upper half only, or its lower half only, may also be present. Any combination of arrowheads — barbed, blunted, curved, straight, left or right, full, lower half, or upper half — may occur.

- (1)  (arrow, left upper barb only)
- (2)  (arrow, left lower barb only)
- (3)  (arrow, right upper barb only)
- (4)  (arrow, right lower barb only)
- (5)  (arrow, upper barbs only)
- (6)  (arrow, lower barbs only)
- (7)  (arrow, left upper barb and right lower barb)
- (8)  (arrow, left lower barb and right upper barb)
- (9)  (arrow, left upper barb and full right barb)
- (10)  (arrow, left lower barb and full right barb)
- (11)  (arrow, full left barb and right upper barb)
- (12)  (arrow, full left barb and right lower barb)
- (13)  (arrow, full left and right barbs)

RULE XXII—MISCELLANEOUS SIGNS AND SYMBOLS

Angstrom Unit	$\text{\AA}$	
At	@	
Caret (circumflex)	$\wedge$	
Cent	¢	
Check Mark	✓	
Crossed d	đ	
Crossed h	ħ	
Crossed Lambda	λ	
Crossed R	Ŕ	
Degree	°	
Del (nabla, gradient)	∇ or ∇	
Ditto Mark	"	
Dollar	\$	
Empty Set		
Represented by Zero with Vertical or Oblique Bar Through It	ϕ or ø	
Represented by Facing Braces	{ }	
Factorial	!	
Infinity	∞	
Integral		
Single	$\int$	
Double	$\iint$	

Triple	$\int\int\int$	
Lower	$\int$	
Upper	$\overline{\int}$	
Integral with Superposed Circle	$\oint$	
Integral with Superposed Infinity	$\int^\infty$	
Integral with Superposed Rectangle	$\int_{\square}$	
Integral with Superposed Square	$\int_{\blacksquare}$	
Partial Derivative (round d)	∂	
Percent	%	
Pound (sterling)	£	
Prime	'	
Quantifiers		
Existential Quantifier		
There exists, for some	$\exists$ or $\exists$	
There exists uniquely for exactly one	$\exists!$ or $\exists!$	
Universal Quantifier (for all, for each, for every)		
	$\forall$ or $\forall$	
Since (because)	$\therefore$	
Tally		
Therefore		
Normal	$\therefore$	
Negated (it does not follow that)	$\ntherefore$	
Vertical Bar (end of proof)		

(1) $1/10,000 \mu = 1 \text{ \AA}$

(1) 3 boxes @ 27¢

[illegible]

(1) 10¢ 

(2) x¢ 

(3) \$2.98 

(4) \$x 

(5) 7% 

(6) x% 

(7) £5 

(8) £x 

(1)  milk     

(2)  eggs       

(3)   bread        

(1) $R(P_1P_2, P_3P_4)$

$$(2) \quad \frac{\partial}{\partial x} f(x, y)$$

$$(3) \quad \frac{\partial^2 u}{\partial x \partial y} =$$

(1) $90^\circ + 90^\circ = 180^\circ$

(1) $\nabla u + \nabla v$ 

(1) 2 goes into 2, 1          

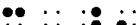
" " " 4, 2          

" " " 6, 3          

(1) $A \cup \emptyset' = A$

(2) $\{\text{even integers}\} \cap \{\text{odd integers}\} = \{\}$

The Braille representation of the set intersection of even and integers and odd integers is shown. It consists of a sequence of Braille cells representing the set notation and the intersection operation. The first row contains the opening curly brace, the words "even integers" and "odd integers" in Braille, the intersection symbol, and the closing curly brace. The second row contains the empty set notation, which is a Braille cell with a single dot 5.

- (3) x''' 
- (4) x'^2 
- (5) x'_1 
- (6) x'_1 
- (7) $(u + v)' = u' + v'$ 
- (8) $\overline{x'}$ 
- (9) $5'8''$ 
- (10) $20^\circ 30' 10''$ 

\$173. Quantifiers: The existential and universal quantifiers must be unspaced from the quantities to which they apply.

- (1) $\exists x, x < \frac{1}{n}$
- (2) $\exists! x \mid x = -x$
- (3) $\forall x \in A$
- (4) $\forall x \forall y \frac{y-x}{x+y} = \frac{x-y}{x+y}$

§174. Since, Therefore: Except for punctuation, indicators, and grouping symbols, the symbol for *since* and symbols for *therefore*, in its normal or negated form, must be spaced from the material to which they apply.

- (1) $\therefore AB = AC$   
- (2) $\therefore AB = AC$   

§175. Tally Marks: Tally marks must be grouped in braille as they are grouped in ink print. However, the cross tally which sometimes appears in ink print must be treated as just another tally mark. Groups of tally marks must be separated by a single space from each other and, except for punctuation, indicators, and grouping symbols, from surrounding material. However, transition to another braille line takes the place of this required space. Transition to another line of braille must never be made from one tally mark to another within the same group.

(1) ||| :: :: ::

(in ink print, there is no cross tally)

(2) ||||| :: :: :: :: :: :: :: :: :: ::

(in ink print, the first group of tallies has a cross tally)

(3) ||||| ||||| || :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: ::

(in ink print, there are no cross tallies)

(4) ||||| ||||| ||||| ||||| ||

:: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: ::

(in ink print, the first four groups of tallies have cross tallies)

§176. **Boldface Vertical Bar:** The single boldface vertical bar meaning *end of proof* must be spaced from any surrounding material.

(1) PROOF. $(b + c) - (a + c) = b - a$ is positive. |

:: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: ::

:: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: :: ::

RULE XXIII—MULTIPURPOSE INDICATOR

Multipurpose Indicator

::

§177. **Use of the Multipurpose Indicator:** The multipurpose indicator must be used in the situations below for the specific purposes described and, when used in these situations, it must not be regarded as the base-line indicator:

i. The multipurpose indicator must be used before a modified expression as an indication to the reader of impending modification. See Rule XIV for additional information and examples.

ii. The multipurpose indicator must be used between a letter and a succeeding numeric symbol to indicate that the corresponding numeral is not a subscript to the corresponding letter. However, when the letter represents a numeral in a numeration system to a base other than 10, it must be regarded as a numeral and, accordingly, the multipurpose indicator must not be used.

(1) x5 :: :: ::

(2) x.6 :: :: :: ::

(3) ≥ 2

(4) T1E4

(a base-12 numeral; in ink print, T and E are capitalized)

iii. The multipurpose indicator must be used between a numeric subscript and a numeral, if the latter is on the base line.

(1) $c_0 10^2 + c_1 10 + c_2$

(2) $2n_1 5^{-3/2} - n_2 5^{-1/2}$

iv. The multipurpose indicator must be used between two symbols of comparison to indicate that the corresponding signs of comparison are printed horizontally and not vertically (see §149).

v. The multipurpose indicator must be used after the decimal point symbol to indicate that the symbol which follows it is not numeric unless that symbol is the comma or the punctuation indicator.

(1) $0.a_1 a_2 \dots$

(2) $0.\alpha_1 \alpha_2 \dots$

(3) $\frac{1}{3} = . \text{---} ?$

(the general omission symbol represents a question mark over a dash in ink print)

(4) $3. + .4 = 3.4$

(5) (3.)

(6) $\frac{1.}{2.}$

vi. The multipurpose indicator must be used between a tally mark and the punctuation indicator.

(1)

(in ink print, the first group of tallies has a cross tally)

vii. The multipurpose indicator must be used between two vertical bars of which the first is a closing grouping symbol and the second is an opening grouping symbol. It must also be used between two vertical bars which are grouping symbols of which one is shorter and/or thicker than the other.

(1) $|x| |y|$

(2) $||x|| ||y||$

(3) $\parallel x \parallel$

(4) $|x|_{x=0}$

viii. The multipurpose indicator must be used between an operation symbol when it is represented by a symbol for a regular polygon and a numeral which follows.

(1) $9 \square 14 = 23$

(2) $9_7 \blacksquare 13_7$

ix. The multipurpose indicator must be used between two symbols for the tilde to indicate that they are written horizontally, one after the other.

(1) $\sim \sim T$

RULE XXIV—SPATIAL ARRANGEMENTS

Division

Curved Division Sign on Left,
Separation Line Above

Curved Division Sign on Right,
Separation Line Above

Curved Division Signs on Left and Right,
Separation Line Above

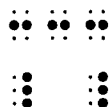
Straight or Slant Division Sign on Left,
Separation Line Above

or

Straight or Slant Division Sign on Right,
Separation Line Above

or

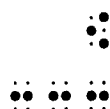
Straight Division Signs on Left and Right,
Separation Line Above



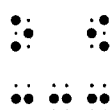
Curved Division Sign on Left,
Separation Line Below



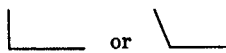
Curved Division Sign on Right,
Separation Line Below



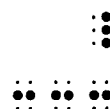
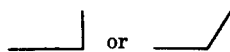
Curved Division Signs on Left and Right,
Separation Line Below



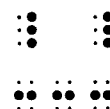
Straight or Slant Division Sign on Left,
Separation Line Below



Straight or Slant Division Sign on Right,
Separation Line Below



Straight Division Signs on Left and Right,
Separation Line Below



Vertical Line Used in Division Arrangements
(varying in length)



Separation Line (varying in length)

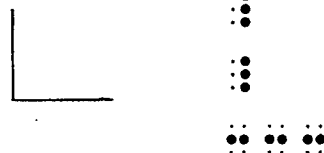


Carried Number Indicator for Addition
(varying in length)

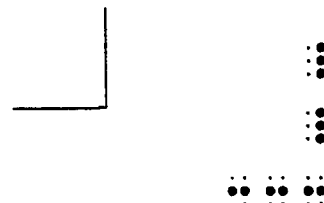


Square Root**Synthetic Division**

**Straight Line on Left, Separation
Line Below**



**Straight Line on Right, Separation
Line Below**

**§178. Addition and Subtraction:**

a. In a spatial arrangement for addition or subtraction, the numeric symbols, fractions, abbreviations, interior signs of operation or comparison must be vertically aligned with digits under digits, commas under commas, decimal points under decimal points, fractions under fractions, abbreviations under abbreviations, signs of operation under signs of operation, and signs of comparison under signs of comparison. However, if these are deliberately misaligned in ink print as in an exercise requiring the student to make a suitable correction, this misalignment must be preserved in the transcription.

b. The plus, minus, or dollar symbols, if the corresponding signs are present, must be placed at least one column of cells to the left of the widest column of numeric symbols which appears in the part of the arrangement *above* the separation line. Subject to the rules above, symbols of operation and dollar symbols may be placed in the same position as shown in ink print.

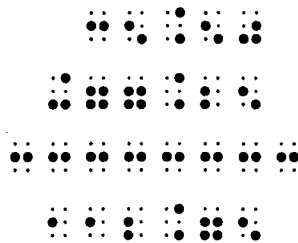
c. The separation line which appears in addition or subtraction must be made one cell longer at either end than the over-all width of the rest of the arrangement.

(1)

508	⠠⠠⠠
2876	⠠⠠⠠⠠
59	⠠⠠
+ 427	⠠⠠⠠⠠
3870	⠠⠠⠠⠠

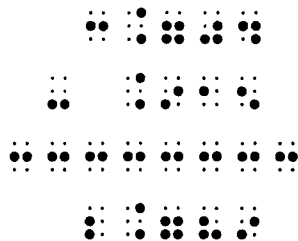
(in ink print, the plus sign is further to the left than any term in the problem or the answer)

$$(2) \quad \begin{array}{r} 35.50 \\ + 77.25 \\ \hline 112.75 \end{array}$$



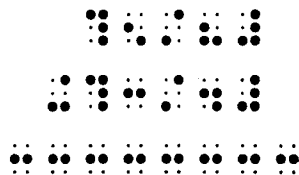
(in ink print, the plus sign is further to the left than any term in the problem or the answer)

$$(3) \quad \begin{array}{r} 3.704 \\ - .915 \\ \hline 2.789 \end{array}$$



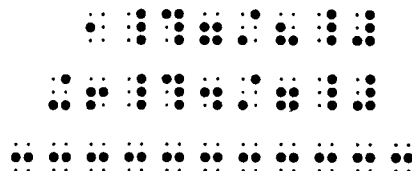
(in ink print, part of the minus sign falls under the 3, the rest extends further to the left)

$$(4) \quad \begin{array}{r} \frac{5}{8} \\ + \frac{3}{4} \\ \hline \end{array}$$



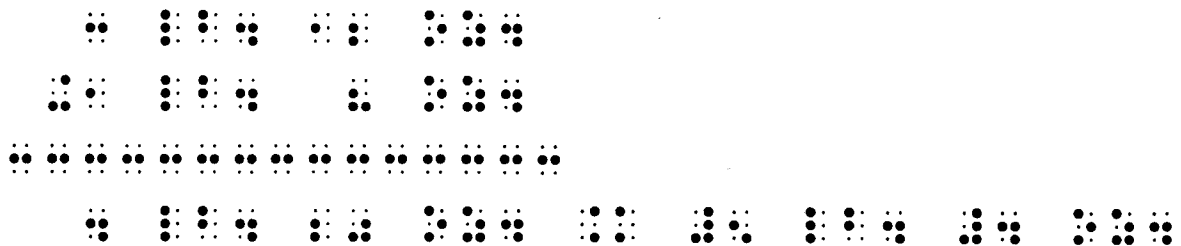
(in ink print, the plus sign is further to the left than any term in the problem)

$$(5) \quad \begin{array}{r} 1 \frac{7}{8} \\ + 6 \frac{4}{7} \\ \hline \end{array}$$



(in ink print, the plus sign is further to the left than any term in the problem)

$$(6) \quad \begin{array}{r} 3 \text{ lb. } 12 \text{ oz.} \\ + 1 \text{ lb. } 8 \text{ oz.} \\ \hline 4 \text{ lb. } 20 \text{ oz.} = 5 \text{ lb. } 4 \text{ oz.} \end{array}$$



(in ink print, the plus sign is further to the left than any term in the problem)

d. **Carried Numbers in Addition:** When carried numbers appear in an addition arrangement above the columns to which they apply, the transcriber must insert the indicator for carried numbers between these carried numbers and the arrangement to which they apply. The carried number indicator must have the same length as the separation line.

(1)

$$\begin{array}{r} \overset{11}{254} \\ +176 \\ \hline 430 \end{array}$$

(in ink print, the carried numbers are in small type directly above the columns to which they apply)

e. In an arrangement containing fractions, fraction lines must be vertically aligned, each numerator must be right justified in the column reserved for numerators, and each denominator must be left justified in the column reserved for denominators. Fraction indicators must also be vertically aligned and must be right-justified in the columns reserved for both opening and closing indicators.

(1)

$$\begin{array}{r} \frac{3}{8} \\ + \frac{4}{8} \\ \hline \frac{7}{8} \end{array}$$

(2)

$$\begin{array}{r} \frac{11}{16} \\ + \frac{1}{2} \\ \hline \end{array}$$

(3)

$$\begin{array}{r} \frac{1}{5} \\ - \frac{1}{10} \\ \hline \end{array}$$

$$\begin{array}{r}
 (4) \quad 1 \frac{5}{6} \\
 + \frac{2}{3} \\
 \hline
 \end{array}$$

f. In an arrangement containing mixed numbers, the whole-number part must be vertically aligned according to a above.

$$\begin{array}{r}
 (1) \quad 10 \frac{2}{3} \\
 + 4 \frac{1}{3} \\
 \hline
 14 \frac{3}{3} = 15
 \end{array}$$

g. In an arrangement containing polynomials, terms must be vertically aligned. In each term, symbols of operation, coefficients, letters, superscript indicators, superscripts, and base-line indicators must also be vertically aligned. When the base-line indicator is required, it must be placed in the first possible position consistent with this required alignment. Within each coefficient and superscript, corresponding symbols must be vertically aligned.

$$\begin{array}{r}
 (1) \quad 2x^3 - x^2 + x + 1 \\
 \quad \quad 3x^3 + 4x^2 - 10x + 7 \\
 \quad \quad \quad 5x^2 \quad + 12 \\
 \hline
 - 2x^3 \quad \quad - 6x \\
 \hline
 3x^3 + 8x^2 - 15x + 20
 \end{array}$$

§179. Multiplication:

a. In a spatial arrangement for multiplication, the symbols comprising the multiplier and multiplicand must be aligned in the transcription as the corresponding signs are aligned in ink print.

b. The multiplication symbol, if the corresponding sign is present in ink print, must be placed immediately to the left of the multiplier.

c. The separation lines which appear in a multiplication arrangement must be made one cell longer at either end than the overall width of the rest of the arrangement.

(1)
$$\begin{array}{r} 23 \\ \times 54 \\ \hline 92 \\ 115 \\ \hline 1242 \end{array}$$

A 10x10 grid of dots representing a sparse matrix. The dots are arranged in a pattern that is symmetric about the main diagonal, which is highlighted by a series of dots from the top-left to the bottom-right. The matrix is sparse, with most cells being empty.

(2)
$$\begin{array}{r} 1704 \\ \times 5 \\ \hline 8520 \end{array}$$

(3)
$$\begin{array}{r} 132 \\ \times 300 \\ \hline 39600 \end{array}$$

(4)
$$\begin{array}{r} 2 \text{ gal } 3 \text{ qt} \\ \times 2 \\ \hline 4 \text{ gal } 6 \text{ qt} \end{array}$$

d. In an arrangement containing fractions, mixed numbers, or polynomials, alignment must be generally as specified in §178 e-g.

(1)

$$\begin{array}{r}
 75 \\
 \times 2\frac{1}{2} \\
 \hline
 56\frac{1}{2} \\
 150 \\
 \hline
 206\frac{1}{2}
 \end{array}$$

(2)

$$\begin{array}{r}
 7x - 3 \\
 4x + 5 \\
 \hline
 28x^2 - 12x \\
 + 35x - 15 \\
 \hline
 28x^2 + 23x - 15
 \end{array}$$

e. In arrangements which show multiplication to non-decimal bases in which subscripts appear, the subscript indicator must be placed in the first possible position consistent with the alignment required for the addition of partial products.

(1)

$$\begin{array}{r}
 34_{\text{seven}} \\
 \times 12_{\text{seven}} \\
 \hline
 101_{\text{seven}} \\
 34_{\text{seven}} \\
 \hline
 441_{\text{seven}}
 \end{array}$$

§180. Division:

a. In a spatial arrangement for division, the symbols comprising the dividend and the partial products and differences must be aligned in the same way as the corresponding signs are aligned in ink print. Symbols in the quotient must be aligned with their corresponding symbols in the dividend unless they are specifically unaligned in ink print.

b. The division symbol must be placed in the cell directly before the dividend or directly after the dividend according as the corresponding division sign occurs in the forward or reverse direction in ink print. The divisor must be placed so that there is no space between it and the division symbol to which it applies. If a quotient also applies to a division symbol by being placed on the same line as the dividend, no space must be left between the quotient and the division symbol to which it applies. If a horizontal line occurs under a divisor as part of a division sign in ink print, this line must be ignored in the transcription.

c. Each separation line which appears in a division arrangement must begin in the column containing a division symbol and must end in the column containing the other division symbol, if the latter appears in ink print. Otherwise, each separation line must end in a cell one column beyond the overall arrangement. However, when the division arrangement contains only a divisor and a dividend, but no quotient and no partial products and differences, the separation line, whether shown above or below the dividend, must be omitted. In this case, the division arrangement must not be regarded as spatial. In particular, a blank line must not be left above or below such a division arrangement, and the numeric indicator must be used in the appropriate place.

(1) $\begin{array}{r} 8 \\ 6 \overline{)48} \end{array}$

(2) $\overline{648}(8)$

(3) $\frac{6 \overline{)636}}{106}$

(4) $\frac{106}{6 \overline{)636}}$

(5)

$$\begin{array}{r}
 644 \\
 4 \overline{)2576} \\
 \underline{24} \\
 17 \\
 \underline{16} \\
 16 \\
 \underline{16} \\
 0
 \end{array}$$

(6)

$$\begin{array}{r}
 x + 5 \overline{) x^2 + 11x + 30} \\
 \underline{x^2 + 5x} \\
 6x + 30 \\
 \underline{6x + 30} \\
 0
 \end{array}$$

(in ink print, it is clear that the quotient is aligned with the dividend, and there is a horizontal line under the divisor)

$$(7) \quad \begin{array}{r} x + 4 \overline{) x^2 + 12x + 32} \\ \underline{x^2 + 4x} \\ 8x + 32 \\ \underline{8x + 32} \\ 0 \end{array}$$

(in ink print, the quotient is clearly not aligned with the terms in the dividend)

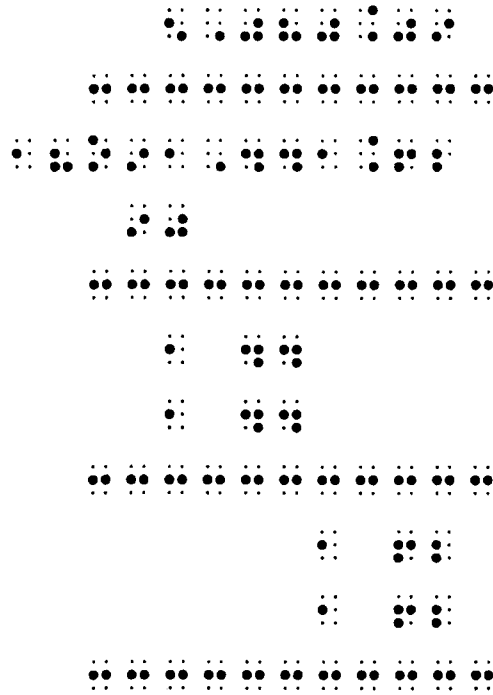
$$(8) \quad 18 \overline{) 452}$$

d. When commas or the decimal point occur in the dividend of a division arrangement, a blank column of cells must be left where these occur in the entire arrangement except in separation lines. When a caret occurs in a dividend, a blank column of two cells must be left where this occurs in the entire arrangement except in the separation lines and the quotient. In the quotient, the decimal point corresponding to the caret must be right-justified in the two cells allotted to the caret.

$$(1) \quad \begin{array}{r} \$ 5.00 \\ 5 \overline{) \$25.00} \end{array}$$

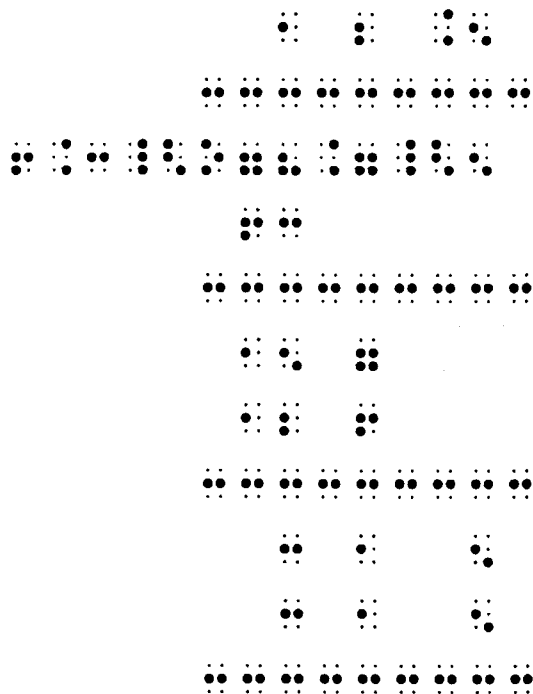
(2)

$$\begin{array}{r}
 5,080.09 \\
 18 \overline{) 91,441.62} \\
 \underline{90} \\
 144 \\
 \underline{144} \\
 162 \\
 \underline{162}
 \end{array}$$



(3)

$$\begin{array}{r}
 12.5 \\
 6.3 \overline{) 78.75} \\
 \underline{63} \\
 157 \\
 \underline{126} \\
 315 \\
 \underline{315}
 \end{array}$$



e. When, in a division arrangement, there is a remainder which is identified as such by the letter "r", lower-case or capitalized, the "r" must be preceded by a space.

(1) 181 r4

25)4529

25

202

200

29

25

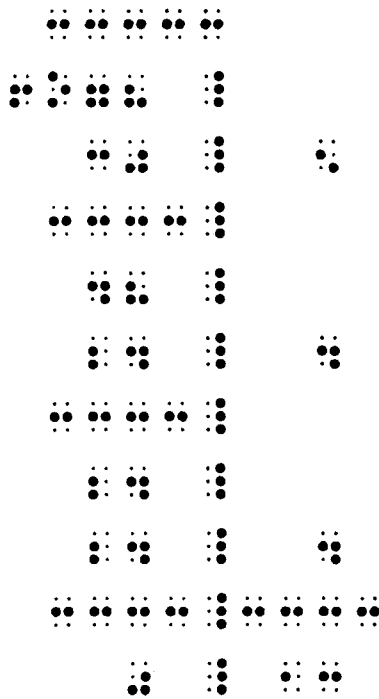
4

181 r4
 25)4529
 25
 202
 200
 29
 25
 4

(1)	6	414	
		60	10
		354	
		120	20
		234	
		180	30
		54	
		54	9
			69

A 10x10 grid of dots forming a sparse pattern. The dots are arranged in a way that suggests a larger, more complex pattern if the grid were extended. The pattern is symmetric about the vertical center line.

$$\begin{array}{r}
 (2) \quad 6 \overline{)78} \\
 \underline{30} \quad 5 \\
 48 \\
 \underline{24} \quad 4 \\
 24 \\
 \underline{24} \quad 4 \\
 0 \quad 18
 \end{array}$$



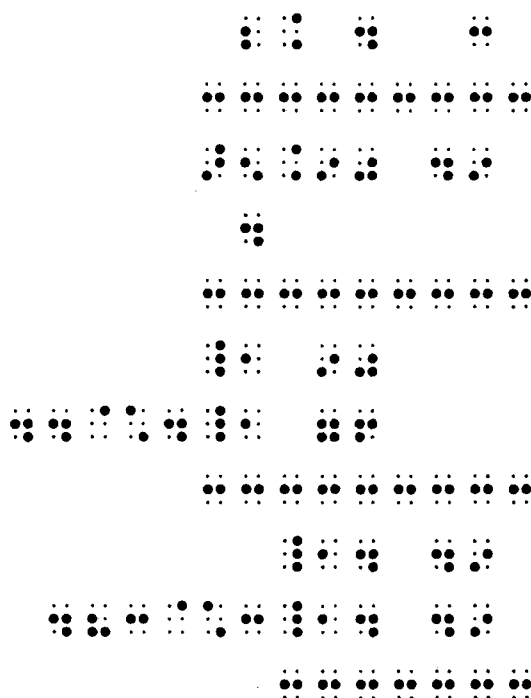
§181. Square Root: A square root arrangement is similar to a division arrangement except that no divisor is present. The arrangement should be adapted to resemble the arrangement in ink print as closely as possible. If the square root symbol is used in this situation, the termination indicator is not required.

(1)

$$\begin{array}{r}
 5.48 \\
 \sqrt{30.00\ 00} \\
 25 \\
 \hline
 104 \quad 5\ 00 \\
 \times 4 \quad 4\ 16 \\
 \hline
 1087 \quad 84\ 00 \\
 \times 7 \quad 76\ 09 \\
 \hline
 7\ 91
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{cccc}
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot
 \end{array} \\
 \begin{array}{cccccccccccc}
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot
 \end{array} \\
 \begin{array}{cccc}
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot
 \end{array} \\
 \begin{array}{cc}
 \cdot\cdot & \cdot\cdot \\
 \cdot\cdot & \cdot\cdot
 \end{array} \\
 \begin{array}{cccccccccccc}
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot
 \end{array} \\
 \begin{array}{cccc}
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot
 \end{array} \\
 \begin{array}{cccc}
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot
 \end{array} \\
 \begin{array}{cccccccccccc}
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot
 \end{array} \\
 \begin{array}{cccc}
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot
 \end{array} \\
 \begin{array}{cccc}
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot
 \end{array} \\
 \begin{array}{cccccccccccc}
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot
 \end{array} \\
 \begin{array}{cc}
 \cdot\cdot & \cdot\cdot \\
 \cdot\cdot & \cdot\cdot
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (2) \quad \begin{array}{r} 2.43 \\ \sqrt{5.9049} \\ 4 \end{array} \\
 44 \times 4 \quad \begin{array}{r} 190 \\ 176 \\ \hline 1449 \\ 1449 \end{array} \\
 483 \times 3 \quad \begin{array}{r} 1449 \\ 1449 \end{array}
 \end{array}$$

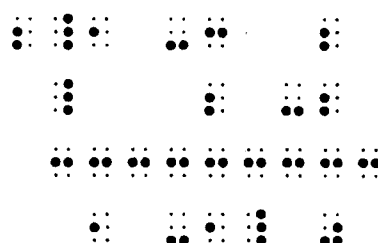


§182. Synthetic Division:

a. In a synthetic division arrangement, the numeric symbols in the synthetic dividend, synthetic product, and synthetic quotient must be aligned by place value. Symbols of operation, when present, must also be aligned. There must be at least one column of blank cells between adjacent columns of a synthetic division arrangement.

b. A vertical line must be used to the left or to the right of the synthetic division arrangement according as the synthetic divisor appears to the left or to the right. This vertical line must be unspaced from the synthetic dividend and from the synthetic divisor. One part of the vertical bar must appear on the line containing the synthetic dividend, and another part of the line must appear on the line containing the synthetic product. The separation line must begin directly under the vertical line at one end, and terminate one cell beyond the over-all synthetic arrangement at the other end. If the synthetic divisor appears in ink print as boxed-in on two sides, this must be ignored in the transcription. When a vertical line is used between the synthetic quotient and the synthetic remainder, it must be placed in the column of blank cells as shown in ink print.

$$(F) \quad 2 \quad \begin{array}{r|rr} 1 & -3 & 2 \\ & 2 & -2 \\ \hline 1 & -1 & 0 \end{array}$$



(2)
$$\begin{array}{r|rr} 1 & -3 & 2 \\ 2 & & -2 \\ \hline 1 & -1 & 0 \end{array} \quad \begin{array}{cccc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}$$

(3)
$$\begin{array}{r|rrrr} +2 & 1 & +6 & -1 & -30 \\ & & +2 & +16 & +30 \\ \hline & 1 & +8 & +15 & +0 \end{array} \quad \begin{array}{cccccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

(in ink print, the divisor is boxed-in on two sides; there is no vertical line after the divisor)

(4)
$$\begin{array}{r|rrrr} 1 & -3 & +4 & +5 \\ +2 & -2 & +4 & \\ \hline 1 & -1 & +2 & +9 \end{array} \quad \begin{array}{cccc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}$$

(in ink print, the divisor is boxed-in on two sides; there is no vertical line after the divisor)

§183. Determinants and Matrices:

a. In determinants or matrices each entry must be left-justified (moved as far left as possible) in the column to which it belongs, and top-justified (moved as far up as possible) in the row to which it applies. Regardless of the ink-print copy, centering or other forms of alignment is not permitted. One column of blank cells must be left between columns.

b. Every effort must be made to confine the entire arrangement to a single braille page. To achieve this goal, the following techniques may be used:

i. An entry may be run over to other braille lines and each continuation indented two cells from the column margin. When space saving is a factor, runovers may be made without regard to any hierarchy preferences. Successive rows in a column must be transcribed without skipping a line between them.

ii. An entry may be run over to other braille lines and each continuation left-justified in its column. When space saving is a factor, runovers may be made without regard to any hierarchy preferences. Successive rows in a column must be transcribed with a

skipped line between them. When the technique described in i above is effective in providing the required space, it must be used in preference to the technique described here.

iii. Additional space may be saved by drawing the enclosing grouping symbols instead of using their braille equivalents.

iv. When an entry is a fraction, the fraction may be represented spatially, if necessary, to save space. However, the row containing such a fraction must then have a line skipped above and below it.

v. The technique of keying may be employed for one or several entries if no other space-saving technique is effective. (See §187.)

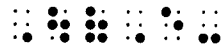
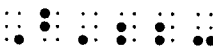
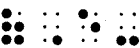
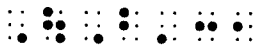
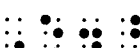
(1)
$$\begin{bmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(in ink print, each entry is centered in the column to which it belongs)

(2)
$$\begin{pmatrix} \frac{1}{A} & 0 & 0 \\ 0 & \frac{1}{A \sin^2 \beta} & -\frac{\cos \beta}{A \sin^2 \beta} \\ 0 & -\frac{\cos \beta}{A \sin^2 \beta} & \frac{1}{C} + \frac{\cos^2 \beta}{A \sin^2 \beta} \end{pmatrix}$$

(in ink print, each entry is centered in the column to which it belongs)

(3)	$B'_{11} - (E - E_1^0)$	B'_{12}	B'_{13}	B'_{14}
	B'_{21}	$B'_{22} - (E - E_2^0)$	B'_{23}	B'_{24}
	B'_{31}	B'_{32}	$B'_{33} - (E - E_3^0)$	B'_{34}
	B'_{41}	B'_{42}	B'_{43}	$B'_{44} - (E - E_4^0)$

(in ink print, each entry is centered in the column to which it belongs)

$$(4) \begin{pmatrix} \frac{1}{A} & 0 & 0 \\ 0 & \frac{1}{A \sin^2 \beta} & \frac{\cos \beta}{A \sin \beta} \\ 0 & \frac{\cos \beta}{A \sin^2 \beta} & \frac{\cos^2 \beta}{A \sin^2 \beta} \end{pmatrix}$$

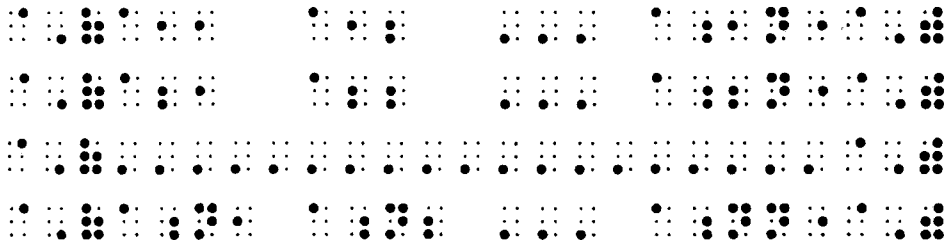
(in ink print, each entry is centered in the column to which it belongs)

c. When a sequence of dots appears to signify the omission of one or more rows and such dots are confined to each column of the determinant or matrix, a sequence of three dots 3 must be placed in each column to indicate the omission and each ellipsis used in this way must be left-justified in its column. When a sequence of dots appears to signify the omission of one or more rows and such dots are not confined to their columns, or if some columns contain no dots, a sequence of dots 3 must be used beginning in the first cell of column one and extending to the end of the longest entry in the last column.

$$(1) \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

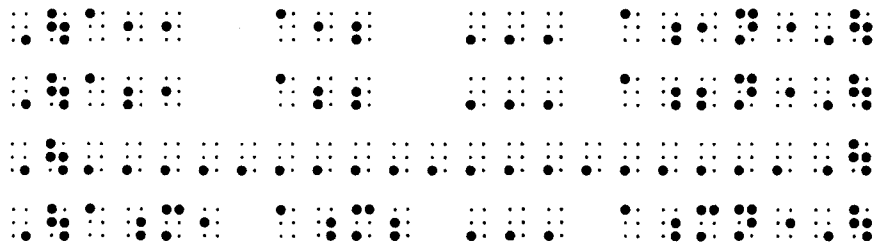
(in ink print, one dot is shown in the first, second, and fourth columns, respectively)

$$(2) \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$



(in ink print, the sequence of dots in the third row is not confined to specific columns)

$$(3) \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

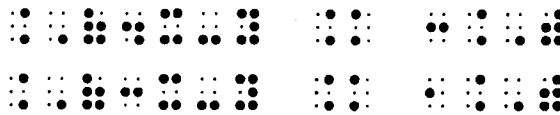


(in ink print, three dots are vertically aligned, one under the other in the first and fourth columns; no such dots appear in the second or third columns)

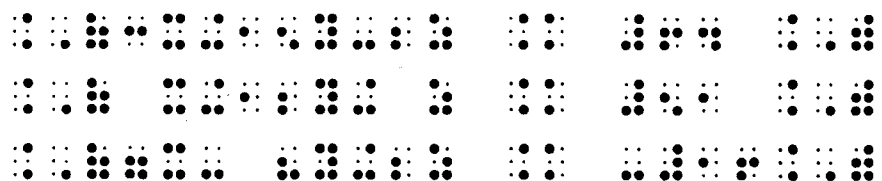
§184. Unified Expressions:

a. When enlarged grouping symbols are used to unify an expression which is neither a determinant nor a matrix, each item must begin in the cell which immediately follows the left enlarged grouping symbol and must end in the cell which immediately precedes the right enlarged grouping symbol. It is advantageous to draw these enlarged grouping symbols when space saving is a factor. However, these requirements must be waived whenever vertical alignment must be indicated. In this case, at least one item must either begin in the cell which immediately follows the left enlarged grouping symbol or must end in the cell which immediately precedes the right enlarged grouping symbol.

$$(1) \left\{ \begin{array}{l} 4x - y = 3 \\ 3x - y = 1 \end{array} \right\}$$



$$(2) \left\{ \begin{array}{l} 3x + 15y - 2z = 64 \\ x + 12y + z = 51 \\ 7x - 8y + 2z = -16 \end{array} \right\}$$



(1) 1. 4956
789
+ 31

(2) 2. \$18.24
 × 65
 —
 9120
 10944
 —
 \$1,185.60

(3) 3. $\begin{array}{r} \overset{1}{2}7 \\ + 5 \\ \hline 32 \end{array}$

(4) 4.
$$\begin{array}{r} 2 \quad 16 \quad 16 \\ \cancel{8} \quad \cancel{7} \quad \cancel{8} \\ - 1 \quad 9 \quad 8 \\ \hline 1 \quad 7 \quad 8 \end{array}$$

ii. The identifier must be placed on the line which contains the dividend in a division arrangement, on the line which contains the radicand in a square root arrangement, and on the line which contains the synthetic dividend in a synthetic division arrangement.

(1) 5.
$$\begin{array}{r} 4947 \\ 5 \overline{)24735} \end{array}$$

iii. In the case of spatial fractions, identifiers and centered comparison symbols, symbols of operation, punctuation, and other applicable symbols must be placed on the principle fraction line. However, identifiers must be placed on the top line of a continued fraction.

(1) 6.
$$\frac{1}{2} + \frac{3}{4} = 1\frac{1}{4}$$

iv. In the case of determinants, matrices, and unified expressions, identifiers, comparison symbols, symbols of operation, punctuation, and other applicable symbols must, if they appear on the same side of the expression as the enlarged grouping symbol, be placed on the top line even though they are centered in ink print.

(1) 7.
$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 100$$

(in ink print, the material outside the determinant is centered)

(2)

$$8. \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 10 \\ 3 \end{bmatrix} .$$

(in ink print, the example number, the multiplication dot, the second matrix, the equals sign, and the period are all centered vertically in relation to the first and last matrices)

(3)

$$9. (-1 \ 4 \ 2) \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} .$$

(in ink print, the example number, first matrix, and period are vertically centered in relation to the second matrix)

(4)

$$10. \begin{cases} x + 3y + z = 5 \\ 2x + y + 2z = 5 \\ 7x + 8y + z = 7 \end{cases} .$$

(in ink print, the example number and the period are vertically centered in relation to the unified system)

c. When spatial arrangements are placed side-by-side there must be at least one clear column of blank cells between the end of one separation line and the beginning of the next. In any case, no symbol in one spatial arrangement may be less than three cells distant from any symbol on any line in, or associated with, a neighboring arrangement other than neighboring ends of separation lines.

(1)

27	15		
+ 7	+23		

$\begin{array}{cccccccccccccccc} \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \end{array}$

(in ink print, the examples are side-by-side)

(2)

1. 42	2. 100
—23	— 91

$\begin{array}{cccccccccccccccc} \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \end{array}$

(in ink print, the examples are side-by-side)

§186. Transcriber's Notes:

a. Transcriber's notes must be enclosed by the transcriber's grouping symbols.

b. A transcriber's note consisting of seven words or less may be inserted directly into the text at the point where it applies. Longer notes must be placed at the nearest convenient point relative to the material to which they apply and must be placed, indented, and run over in accordance with the rules of the *Code of Braille Textbook Format and Techniques*.

(1) In x^2 , the 2 is the exponent.

$\begin{array}{cccccccccccccccc} \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \end{array}$

§187. Keying Technique:

a. When space does not permit the inclusion of labels, column headings, entries, etc., in a figure, determinant, matrix, or table as shown in ink print, one or more of the labels, headings, entries, etc. may be replaced by a numeric or alphabetic key. A numeric key should consist of a numeral written in the upper part of the braille cell. This numeral must be preceded by the numeric indicator and must not be punctuated. An alphabetic key must consist of two lower-case English letters and, if possible, the combination should be suggestive of the item it represents. An alphabetic key may only be used when the author's entries are never composed of two lower-case letters. Two items which are identical should have the same key assigned to them.

b. If a list of numeric keys is used, it must consist of consecutive numerals beginning with number 1, and these numerals should be placed in the figure, determinant, matrix, or table in the same position as the material which they replace.

c. A list of numeric or alphabetic keys and their meanings must be enclosed in transcriber's grouping symbols and must precede the material to which it applies. Key items may be arranged vertically at the margin, or they may be arranged in columns to save space. A key listing must be preceded and followed by a blank line. If possible, this list must be placed on the same braille page as the material to which it applies.

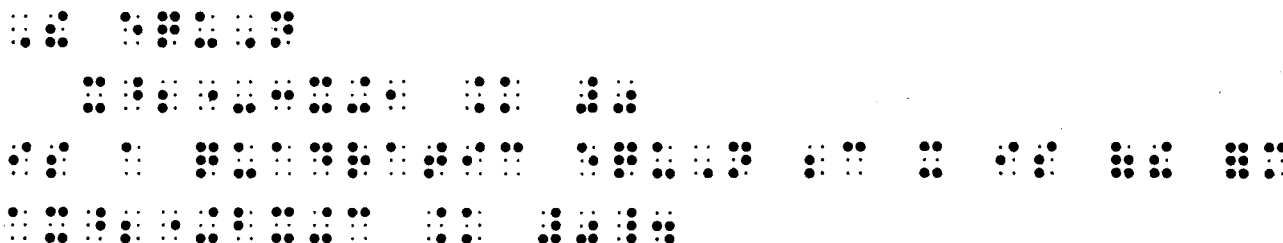
(1)

Source of Variation	Sums of Squares	D.F.	Mean Square	EMS	F-Ratio
Between blocks	$SS_i = 2$	2	$\frac{2}{2} = 1$	$\sigma^2 + \frac{3}{2} \sum \alpha_i^2$	$\frac{1}{2.5} = .4$
Between treatments	$SS_j = 26$	2	$\frac{26}{2} = 13$	$\sigma^2 + \frac{3}{2} \sum \beta_j^2$	$\frac{13}{2.5} = 5.2$
Error	$SS_{ij} = 10$	4	$\frac{10}{4} = 2.5$	σ^2	
Total	$SS = 38$	8			

- (1) The equation

$$x^2 - 3x + 1 = 0$$

is a quadratic equation because it is of the form $ax^2 + bx + c = 0$.



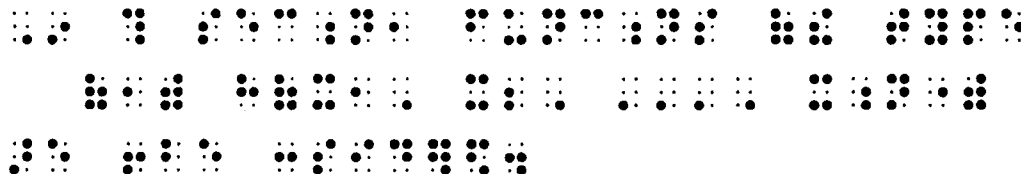
(the first equation is a displayed expression; the second equation is an embedded expression)

b. If an identifying number or letter is associated with a displayed expression, this number or letter is part of the displayed expression and must begin in the appropriate cell in accordance with the rules for displayed expressions in §§190-191. In ink print, identifying numbers or letters are sometimes at the right. In braille, numbers or letters must be placed uniformly at the left. However, if identifying numbers or letters occur at the right in ink print, a transcriber's note concerning the transposition of such numbers or letters must be placed at the beginning of the first volume. Page references which are associated with an expression must immediately follow that expression.

- (1) In this section, functions of the type

$$h(x_1, x_2, \dots, x_n) \quad (1)$$

are to be considered.

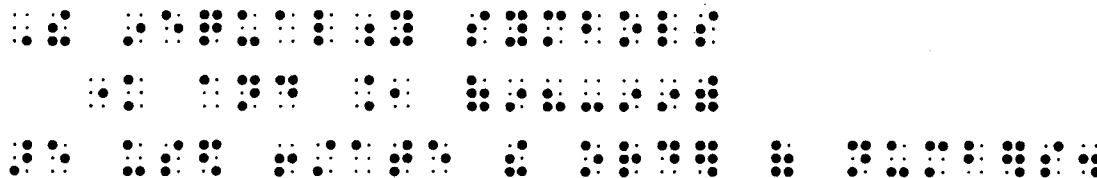


(in ink print, the identifying number occurs at the right)

- (2) The inequality symbols

$$< \quad \text{and} \quad > \quad (98-99)$$

are used to state the order of numbers.



(in ink print, the page numbers occur at the right-hand margin of the page)

§189. Linked Expressions:

a. A linked expression must contain at least one sign of comparison. The component which precedes the first sign of comparison is called the *anchor*. Each of the remaining components, beginning with a comparison sign but not including the next comparison sign, is called a *link*.

b. When the special margin requirements for linked expressions do not apply, a displayed expression must begin in cell 3 and must be run over, if necessary, in cell 5.

- (1) The product of two monomials is a monomial.

For example

$$(3x^2)^3 = (3x^2)(3x^2)(3x^2) = (9x^4)(3x^2) = 27x^6.$$

c. When the special margin requirements for linked expressions do apply, the anchor must begin in cell 3 and must be run over, if necessary, in cell 7. Each link must begin in cell 5 on a new braille line and must be run over, if necessary, in cell 7.

- (1) $8x^3 + 125y^3$ can be factored in the following way:

$$\begin{aligned} 8x^3 + 125y^3 &= (2x)^3 + (5y)^3 \\ &= (2x + 5y)[(2x)^2 - (2x)(5y) + (5y)^2] \\ &= (2x + 5y)(4x^2 - 10xy + 25y^2). \end{aligned}$$

(in ink print, the first equals sign appears to the right of $8x^3 + 125y^3$ and all other equals signs are aligned beneath it)

§191. Margins for Non-Spatial Itemized Materials: When material is identified sequentially by number or letter, as in exercises or outlines, it will be referred to as *itemized material*.

a. When non-spatial itemized material contains main divisions only (no subdivisions) the following rules concerning margins must be observed:

i. The main division numbers or letters must begin in cell 1 and the associated material must be run over, if necessary, in cell 3.

ii. Succeeding paragraphs, if any, must begin in cell 5 and must be run over, if necessary, in cell 3.

iii. When the special margin requirements for linked expressions do not apply, a displayed expression must begin in cell 5 and must be run over, if necessary, in cell 7.

iv. When the special margin requirements for linked expressions do apply, the anchor must begin in cell 5 and must be run over, if necessary, in cell 9. Each link must begin in cell 7 and must be run over, if necessary, in cell 9.

v. Instructions which apply to a group of problems which follow must begin in cell 5 and must be run over, if necessary, in cell 3. There must be a blank line above such instructions, but not below. However, a page-change line may take the place of this required skipped line. The last line of an instruction and the first line of a problem to which it applies must be on the same braille page.

- (1) 1. Is $(y - 3)$ a factor of $y^3 + 3y^2 - 7y - 33$? If so, what is the other factor?

Check by division, or as shown in Chapter 9.

$$\begin{array}{r}
 y^3 + 3y^2 - 7y - 33 \\
 \underline{-(y^3 - 3y^2 + 7y - 33)} \\
 6y^2 - 14y \\
 \underline{-(6y^2 - 18y + 33)} \\
 4y - 33 \\
 \underline{-(4y - 12)} \\
 -21
 \end{array}$$

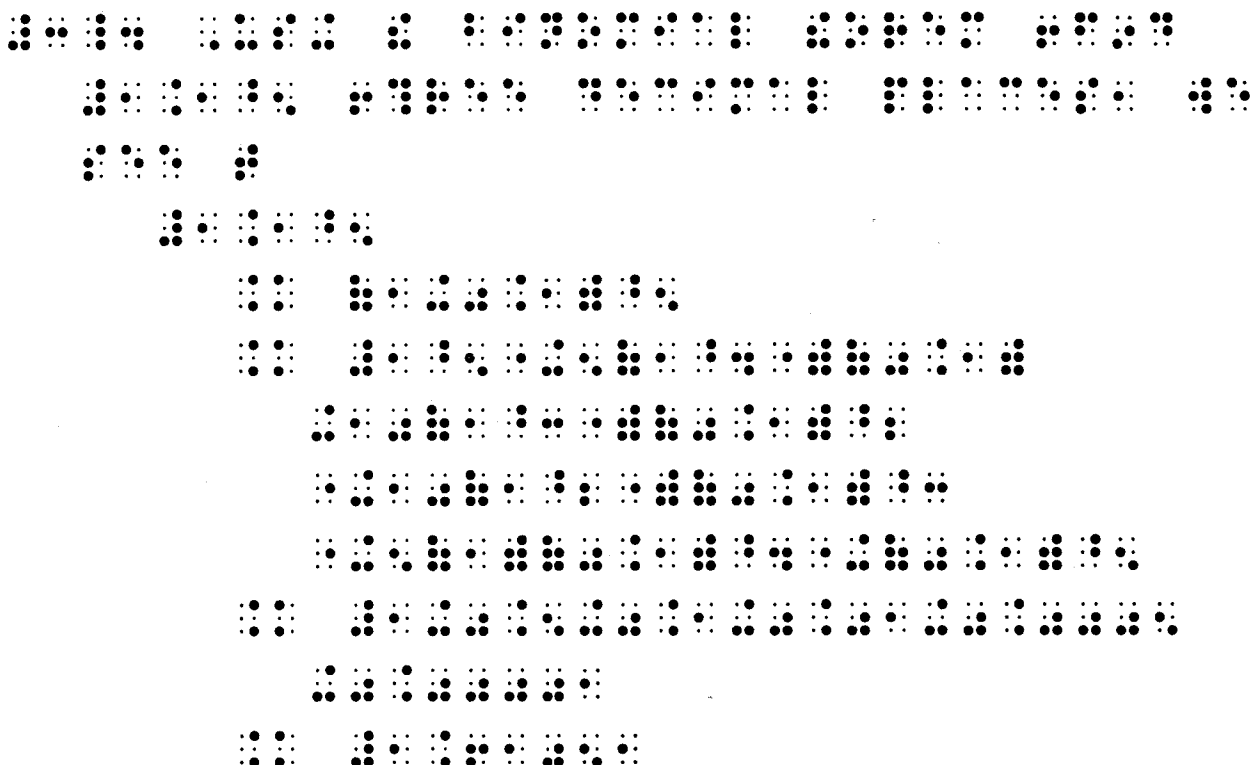
- (2) 2. Write the single numeral that names the same number as

$$(3 \times 10^4) + (4 \times 10^3) + (5 \times 10^2) + (6 \times 10) + (7 \times 1).$$

$$\begin{array}{r}
 30000 + 4000 + 500 + 60 + 7 \\
 = 34567
 \end{array}$$

- (3) 3. Using the binomial theorem to find 1.1^5 to three decimal places, we see that

$$\begin{aligned}
 1.1^5 &= (1 + 0.1)^5 \\
 &= 1^5 + 5(1^4)(0.1) + 10(1^3)(0.1)^2 + 10(1^2)(0.1)^3 + \\
 &\quad 5(1)(0.1)^4 + (0.1)^5 \\
 &= 1 + 0.5 + 0.1 + 0.01 + 0.0005 + 0.00001 \\
 &= 1.61051
 \end{aligned}$$



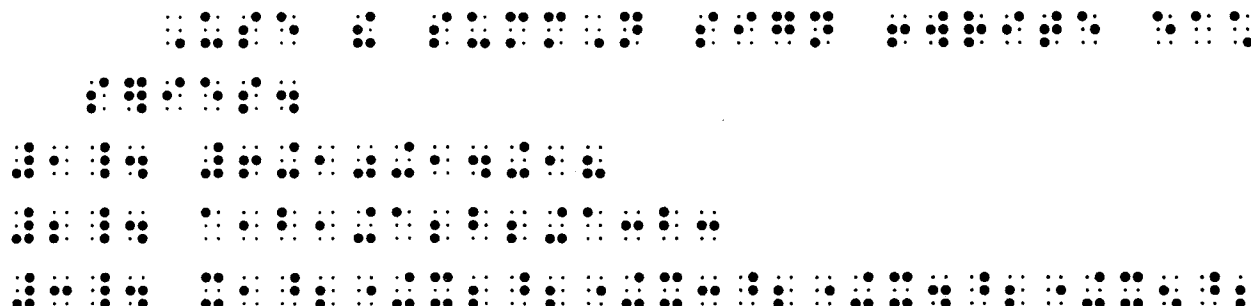
(in ink print, the first equals sign is to the right of 1.1⁵ and all other equals signs are aligned beneath it)

(4) Use the summation sign to write each series.

1. $6 + 10 + 14 + 18$

2. $a_1b_1 + a_2b_2 + a_3b_3$

3. $x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2$



b. When non-spatial itemized material contains both main divisions and subdivisions to whatever depth, the following rules concerning margins must be observed:

- i. The main division numbers or letters must begin in cell 1 and the associated material must be run over, if necessary, in cell 5.
- ii. Subdivision numbers or letters, regardless of depth, must begin in cell 3 and must be run over, if necessary, in cell 5.

iii. Succeeding paragraphs, if any, must begin in cell 7 and must be run over, if necessary, in cell 5.

iv. When the special margin requirements for linked expressions do not apply, a displayed expression must begin in cell 7 and must be run over, if necessary, in cell 9.

v. When the special margin requirements for linked expressions do apply, the anchor must begin in cell 7 and must be run over, if necessary, in cell 11. Each link must begin in cell 9 and must be run over, if necessary, in cell 11.

vi. Instructions which apply to a group of problems which follow must begin in cell 5 and must be run over, if necessary, in cell 3. There must be a blank line above such instructions, but not below. However, a page-change line may take the place of this required skipped line. The last line of an instruction and the first line of a problem to which it applies must be on the same braille page.

- (1) 1. Find the replacement for N that will make each sentence true.

a. $(3 \times 5) \times 2 = 3 \times (N \times 2)$

b. $3 \times (5 \times 2) = (3 \times 5) \times N$

Did you use the same numeral as a replacement in each sentence? Is this sentence true:

$(3 \times 5) \times 2 = 3 \times (5 \times 2) ?$

Braille representation of the text above, including the equations and the question.

- (2) 2. a. $x(a + 1) - y(a + 1)$

b. $x^2 - 2x + 1 - 4a^2 - 12a - 9$

Braille representation of the text above, including the equations.

(in ink print, the a is on the same line as the problem number, and the b is aligned beneath a)

(3) 3. In factoring $ab + c^2 + ac + bc$:

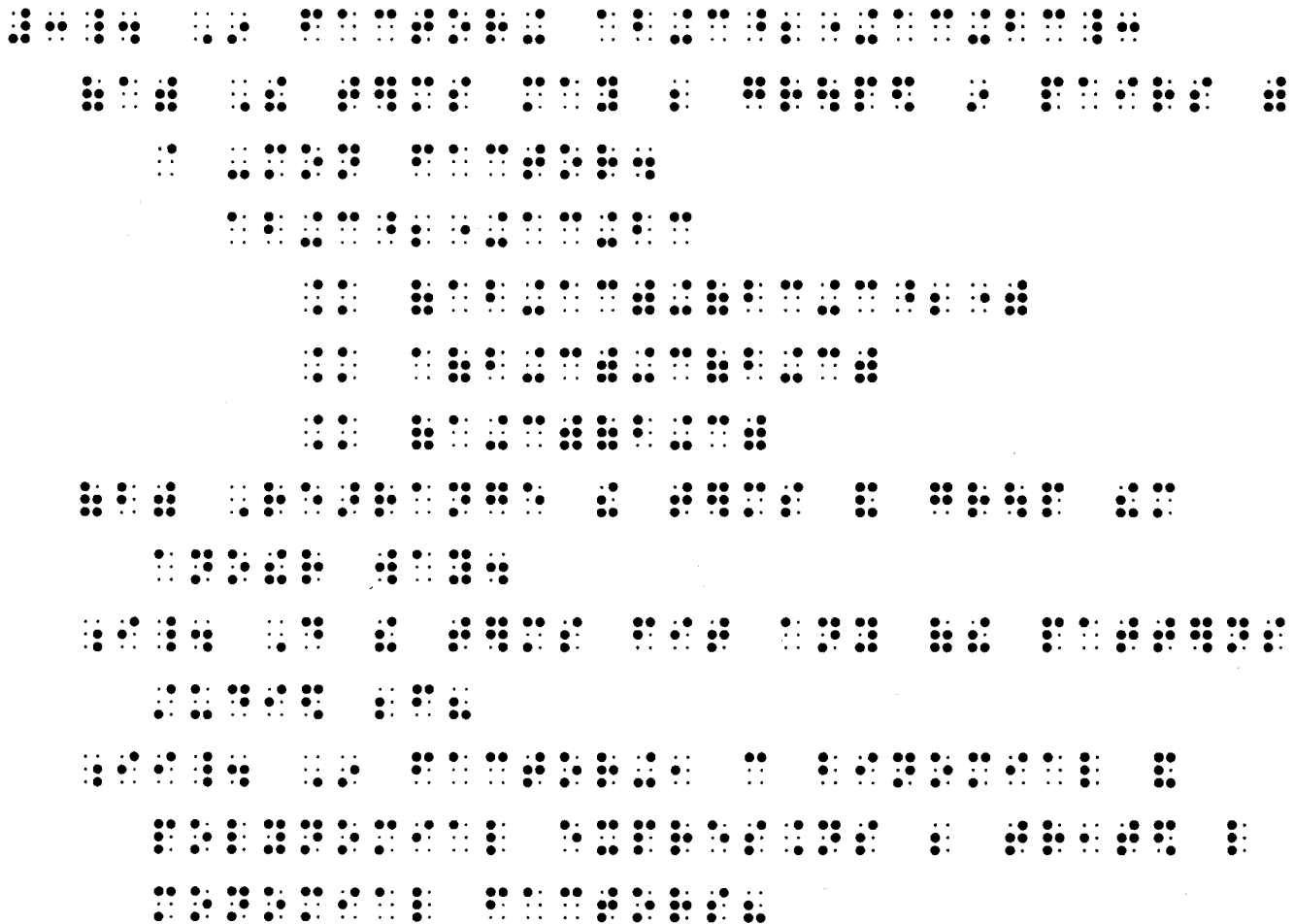
(a) The terms may be grouped in pairs with a common factor.

$$\begin{aligned} ab + c^2 + ac + bc &= (ab + ac) + (bc + c^2) \\ &= a(b + c) + c(b + c) \\ &= (a + c)(b + c) \end{aligned}$$

(b) Rearrange the terms and group them another way.

i. Do the terms fit any of the patterns studied before?

ii. In factoring, can binomial and polynomial expressions be treated like monomial factors?



(in ink print, the main problem number, the (a) and the (b) are vertically aligned; the Roman numerals are indented further to the right)

(4) Add. Check your addition by adding the other way.

4. a. $118 + 37 + 66$ b. $123 + 159 + 92$

c. $146 + 192$

Braille representation of the addition problems and their solutions:

$118 + 37 + 66 = 211$
 $123 + 159 + 92 = 374$
 $146 + 192 = 338$

(in ink print, the problem number and the first two subdivisions are on the same line; the third subdivision is beneath the first)

c. When non-spatial itemized material contains both main divisions and subdivisions, it is permissible to place all subdivisions on a single braille line if that braille line can accommodate all the subdivisions.

(1) 1. Subtract:

(a) $10 - 3$ (b) $15 - 4$ (c) $21 - 19$

Braille representation of the subtraction problems and their solutions:

$10 - 3 = 7$
 $15 - 4 = 11$
 $21 - 19 = 2$

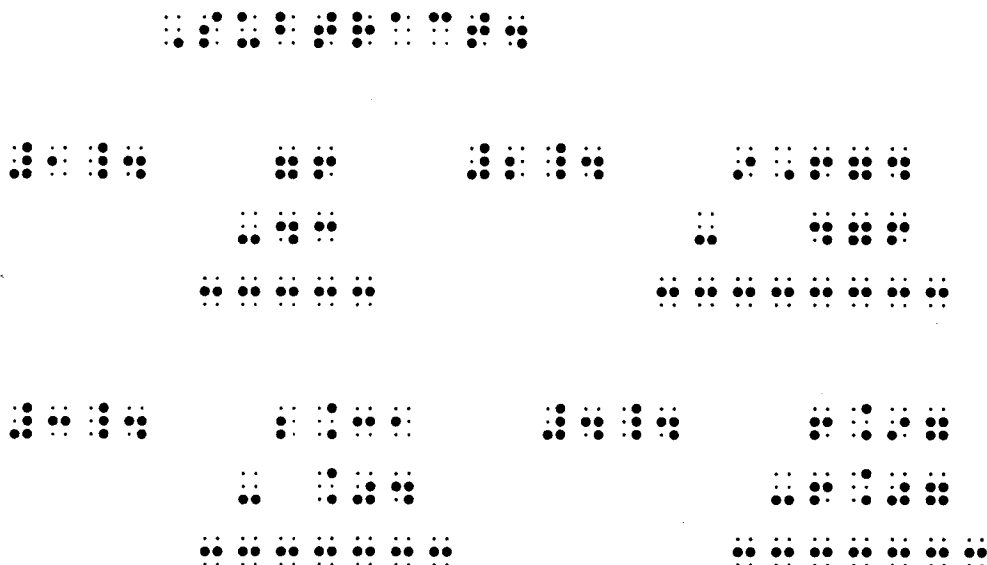
(the braille and ink print are the same)

§192. **Margins for Spatial Itemized Materials:** Spatial itemized material may be transcribed using the same margin rules as are contained in §191. However, for space-saving purposes the following alternatives are available:

a. When spatial itemized material contains main divisions only (no subdivisions), the first division number begins in cell 1. Subsequent division numbers may begin to the right of the preceding spatial arrangement regardless of how they occur in ink print. As many main division numbers and their associated spatial arrangements may occur across the page as can be accommodated. If additional main division numbers remain, the first of these begins again in cell 1, after having left a blank line below the longest of the spatial arrangements which occur above.

(1) Subtract.

1.	76	3.	2.31
	— 43		— .04
2.	9,674	4.	6.97
	— 476		— 6.07



(in ink print, examples 1 and 2 appear in the first column, examples 3 and 4 appear in the second column)

b. When spatial itemized materials contain both main divisions and subdivisions, the first main division number begins in cell 1 and the first subdivision follows on the same braille line if there is no material between the main division number and the subdivision number. As many additional subdivisions may be transcribed across the line as can be accommodated. If additional subdivisions remain, they are started in cell 3 after having left a blank line below the longest of the spatial arrangements which occur above.

- (1) 1. a. $\begin{array}{r} 462 \\ \times 30 \\ \hline \end{array}$ b. $\begin{array}{r} 1,763 \\ \times 142 \\ \hline \end{array}$ c. $\begin{array}{r} 51.986 \\ \times 7.3 \\ \hline \end{array}$ d. $\begin{array}{r} .67 \\ \times .92 \\ \hline \end{array}$
2. a. $\begin{array}{r} 712 \\ \times 480 \\ \hline \end{array}$ b. $\begin{array}{r} 2,547 \\ \times 3 \\ \hline \end{array}$ c. $\begin{array}{r} 8.69 \\ \times .08 \\ \hline \end{array}$ d. $\begin{array}{r} 200.2 \\ \times 100.0 \\ \hline \end{array}$

Braille representation of the multiplication problems above, showing the spatial arrangement of numbers and operators across multiple lines and columns.

(in ink print, all four subdivisions of each problem appear across the page in columnar form)

(2) 2. Multiply.

a. 94621
567

b. 43290
380

c. 1,000,000
432

Figure 1 shows a 2D grid of 10x10 cells, representing a 1000-cell network. The grid is divided into four quadrants by a vertical line at column 5 and a horizontal line at row 5. The top-left quadrant (columns 1-4, rows 1-4) contains a cluster of black cells. The top-right quadrant (columns 6-9, rows 1-4) contains a cluster of black cells. The bottom-left quadrant (columns 1-4, rows 6-9) contains a cluster of black cells. The bottom-right quadrant (columns 6-9, rows 6-9) contains a cluster of black cells. The central area (columns 5 and 10) is mostly white.

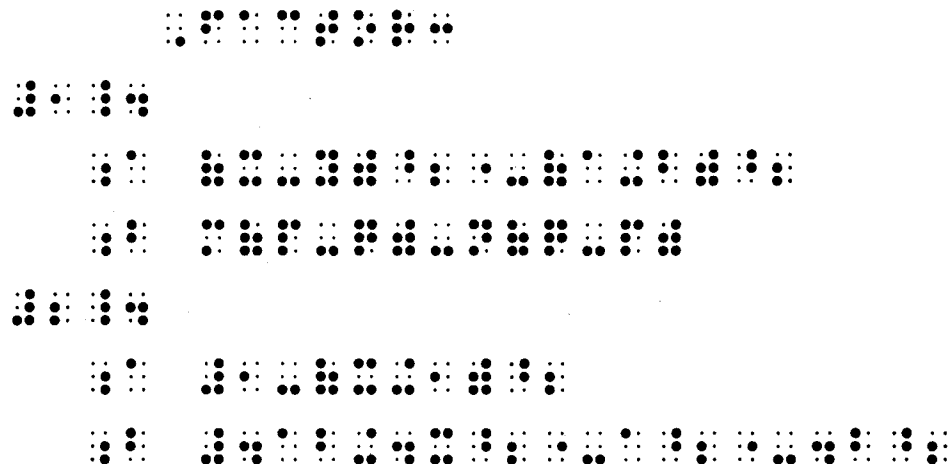
(in ink print, all the subdivisions are on the same line)

§193. Margins for Spatial and Non-Spatial Itemized Materials Arranged in Tabular Form:

- a. When itemized material is arranged in tabular form so that rows are identified by number and columns are identified by letter, the following technique must be used provided that the *entire* tabulation can be contained across the braille page.
 - i. The letters which identify the columns must be left-justified in the columns to which they apply.
 - ii. A blank line must be left above and below the column headings.
 - iii. Row numbers must begin in cell 1.
 - iv. At least two spaces should be left between the right-hand margin of one column and the left-hand margin of the next column.

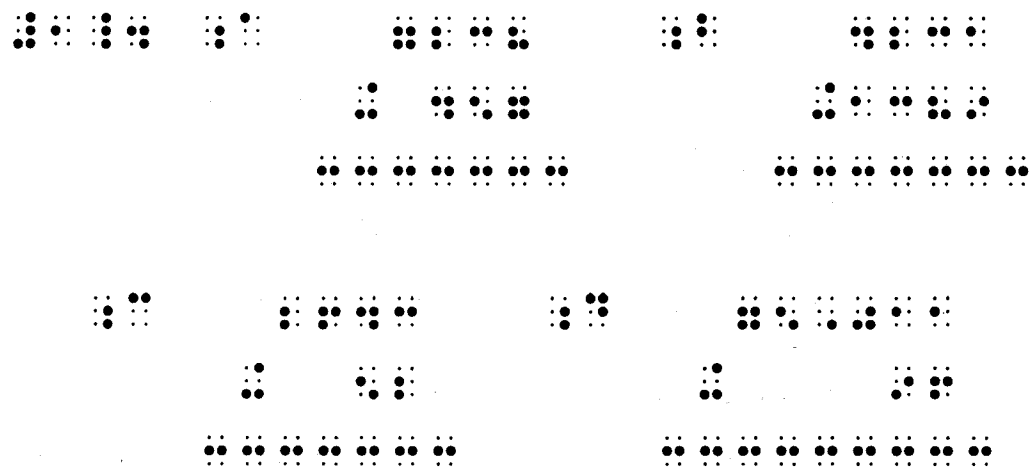
(1) Factor:

	a	b
1.	$(x-y)^2 - (a+b)^2$	$m(p-q) - n(q-p)$
2.	$1 - (x+1)^2$	$4ab + 4x^2 - a^2 - 4b^2$



(in ink print, the subdivisions are placed across the page and are aligned beneath lettered column headings)

	a	b	c	d
1.	7238	4231	2643	75,011
	+ 457	+ 1389	+ 52	+ 96



(in ink print, the subdivisions are placed across the page and are aligned beneath lettered column headings)

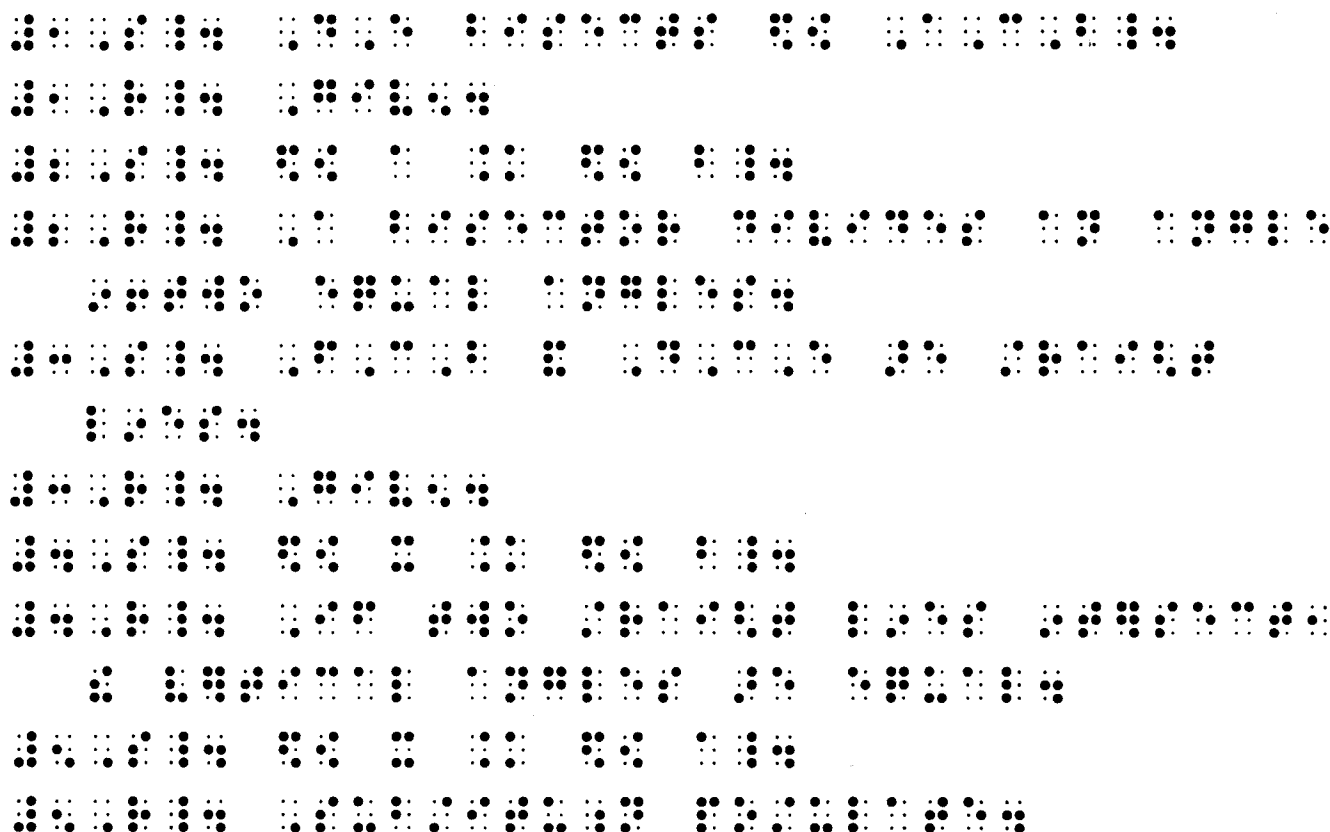
iii. A transcriber's note must be included to call attention to this braille format and to specify the meaning of "S", "R", or other letters which may have been used. This note must be placed at the beginning of each braille volume in which this technique is used.

(1) *Statements*

1. DE bisects $\angle ACB$.
2. $\angle a = \angle b$.
3. FCB and DCE are straight lines.
4. $\angle x = \angle b$.
5. $\angle x = \angle a$.

Reasons

1. Given.
2. A bisector divides an angle into two equal angles.
3. Given.
4. If two straight lines intersect, the vertical angles are equal.
5. Substitution postulate.



(in ink print, there is a Statement column and a Reason column)

- i. After a comma which occurs between items in an "enclosed list."
- ii. Before a symbol of comparison.
- iii. Before a symbol of operation.
- iv. Before a fraction line.
- v. Before the base-line indicator.
- vi. Before a change-of-level indicator or within a superscript or subscript before one of the symbols listed above.
- vii. Between factors which are enclosed within grouping symbols.
- viii. After a termination indicator.

APPENDIX A

COMBINATIONS OF TYPE-FORM, ALPHABETIC, AND CAPITALIZATION INDICATORS

LOWER-CASE LETTERS

Type-form	English letters	German letters	Greek letters	Greek Letter Alternative	Hebrew letters	Russian letters
Boldface	⠠ ⠠	⠠ ⠠	⠠ ⠠			⠠ ⠠ ⠠
Italic	⠠ ⠠	⠠ ⠠	⠠ ⠠			⠠ ⠠ ⠠
Ordinary	⠠ ⠠	⠠ ⠠	⠠ ⠠	⠠ ⠠	⠠ ⠠ ⠠	⠠ ⠠ ⠠
Sanserif	⠠ ⠠					
Script	⠠ ⠠	⠠ ⠠	⠠ ⠠		⠠ ⠠ ⠠	⠠ ⠠ ⠠

CAPITALIZED LETTERS

Boldface	⠠ ⠠ ⠠	⠠ ⠠ ⠠	⠠ ⠠ ⠠			⠠ ⠠ ⠠ ⠠
Italic	⠠ ⠠ ⠠	⠠ ⠠ ⠠	⠠ ⠠ ⠠			⠠ ⠠ ⠠ ⠠
Ordinary	⠠ ⠠ ⠠	⠠ ⠠ ⠠	⠠ ⠠ ⠠			⠠ ⠠ ⠠ ⠠
Sanserif	⠠ ⠠ ⠠					
Script	⠠ ⠠ ⠠	⠠ ⠠ ⠠	⠠ ⠠ ⠠			⠠ ⠠ ⠠ ⠠

APPENDIX B

INDEX OF BRAILLE SYMBOLS

The following is the list of 63 braille symbols arranged in their standard order. The separation of these symbols into the usual seven lines of braille is ignored, but each symbol is numbered in accordance with its rank in the list.

1	⠠	14	⠠⠠	27	⠠⠠⠠	40	⠠⠠⠠⠠	53	⠠⠠⠠⠠⠠
2	⠠⠠	15	⠠⠠⠠	28	⠠⠠⠠⠠	41	⠠⠠⠠⠠⠠	54	⠠⠠⠠⠠⠠⠠
3	⠠⠠⠠	16	⠠⠠⠠⠠	29	⠠⠠⠠⠠⠠	42	⠠⠠⠠⠠⠠⠠	55	⠠⠠⠠⠠⠠⠠⠠
4	⠠⠠⠠⠠	17	⠠⠠⠠⠠⠠	30	⠠⠠⠠⠠⠠⠠	43	⠠⠠⠠⠠⠠⠠⠠	56	⠠⠠⠠⠠⠠⠠⠠⠠
5	⠠⠠⠠⠠⠠	18	⠠⠠⠠⠠⠠⠠	31	⠠⠠⠠⠠⠠⠠⠠	44	⠠⠠⠠⠠⠠⠠⠠⠠	57	⠠⠠⠠⠠⠠⠠⠠⠠⠠
6	⠠⠠⠠⠠⠠⠠	19	⠠⠠⠠⠠⠠⠠⠠	32	⠠⠠⠠⠠⠠⠠⠠⠠	45	⠠⠠⠠⠠⠠⠠⠠⠠⠠	58	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
7	⠠⠠⠠⠠⠠⠠⠠	20	⠠⠠⠠⠠⠠⠠⠠⠠	33	⠠⠠⠠⠠⠠⠠⠠⠠⠠	46	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	59	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
8	⠠⠠⠠⠠⠠⠠⠠⠠	21	⠠⠠⠠⠠⠠⠠⠠⠠⠠	34	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	47	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	60	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
9	⠠⠠⠠⠠⠠⠠⠠⠠⠠	22	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	35	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	48	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	61	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
10	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	23	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	36	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	49	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	62	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
11	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	24	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	37	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	50	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	63	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
12	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	25	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	38	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	51	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠		
13	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	26	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	39	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	52	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠		

The items in the INDEX OF BRAILLE SYMBOLS are "alphabetized" in accordance with the list of the 63 braille symbols above.

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	⠠	German ah 23
	⠠	Greek alpha 23
	⠠	Hebrew aleph 24
	⠠	Russian ah 25
	⠠ ⠠ ⠠	amp (amplitude) 118
	⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠	antilog (antilogarithm) 118
	⠠ ⠠ ⠠	arc (arc) 119
	⠠ ⠠ ⠠	arg (argument) 119
2	⠠ (dots 1-2)	
	⠠	English b 22
	⠠	German beh 23
	⠠	Greek beta 23
	⠠	Russian beh 25
3	⠠ (dots 1-4)	
	⠠	English c 22
	⠠	German tseh 23
	⠠	Greek sampi 23
	⠠	Russian tseh 25

3	⠠⠠ (Cont.)		Page
	⠠⠠⠠⠠⠠⠠⠠⠠	colog (cologarithm)	119
	⠠⠠⠠⠠	cos (cosine)	119
	⠠⠠⠠⠠⠠⠠	cosh (hyperbolic cosine)	119
	⠠⠠⠠⠠	cot (cotangent)	119
	⠠⠠⠠⠠⠠⠠	coth (hyperbolic cotangent)	119
	⠠⠠⠠⠠⠠⠠⠠⠠	covers (coversine)	119
	⠠⠠⠠⠠	csc (cosecant)	119
	⠠⠠⠠⠠⠠⠠	csch (hyperbolic cosecant)	119
	⠠⠠⠠⠠	ctn (cotangent)	119
	⠠⠠⠠⠠⠠⠠	ctnh (hyperbolic cotangent)	119
4	⠠⠠ (dots 1-4-5)		
	⠠⠠	English d	22
		German deh	23
		Greek delta	23
		Russian deh	25
	⠠⠠⠠⠠	det (determinant)	119
5	⠠⠠ (dots 1-5)		
	⠠⠠	English e	22
		German eh	23
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		Russian yeh	25

5	⠠⠠⠠ (Cont.)	Page
	⠠⠠⠠⠠⠠	erf (error function) 119
	⠠⠠⠠⠠⠠	exp (exponential) 119
	⠠⠠⠠⠠⠠⠠⠠	exsec (exsecant) 119
6	⠠⠠⠠⠠ (dots 1-2-4)	
	⠠⠠	English f 22
		German eff 23
		Greek phi 24
		Hebrew feh 24
		Russian eff 25
7	⠠⠠⠠⠠⠠ (dots 1-2-4-5)	
	⠠⠠	English g 22
		German gheh 23
		Greek gamma 23
		Hebrew gimel 24
		Russian gheh 25
	⠠⠠⠠⠠⠠⠠	grad (gradient) 119
8	⠠⠠⠠⠠⠠ (dots 1-2-5)	
	⠠⠠	English h 22
		German hah 23
		Hebrew heh 24

8	(Cont.)	Page
	Russian khah	25
	hav (haversine)	119
9	(dots 2-4)	
	English i	22
	German ee	23
	Greek iota	24
	Russian ee	25
	im (imaginary part)	119
	inf (infimum)	119
10	(dots 2-4-5)	
	English j	22
	German yaht	23
	Hebrew yod	24
	Russian zheh	25
11	(dots 1-3)	
	English k	22
	German kah	23
	Greek kappa	24
	Russian kah	25

12	⠠⠨⠨⠨ (dots 1-2-3)	Page
	English l	22
	German ell	23
	Greek lambda	24
	Hebrew lamed	24
	Russian ell	25
	lim (limit)	119
	ln (natural logarithm)	120
	log (logarithm)	120
13	⠠⠨⠨⠨⠨ (dots 1-3-4)	
	English m	22
	German em	23
	Greek mu	24
	Hebrew mem	24
	Russian em	25
	max (maximum)	120
	min (minimum)	120
	mod (modulo)	120
14	⠠⠨⠨⠨⠨⠨ (dots 1-3-4-5)	
	English n	22
	German en	23

14		(Cont.)	Page
		Greek nu	24
		Hebrew nun	24
		Russian en	25
15		(dots 1-3-5)	
		barbed right full arrowhead	>
		English o	22
		German oh	23
		Greek omicron	24
		Russian oh	25
		curved division sign on left, separation line above	}
			
		curved division sign on left, separation line below	}
			
		curved division signs on left and right, separation line below	)(
			
16		(dots 1-2-3-4)	
		English p	22
		German peh	23

16	⠠⠠⠠ (Cont.)	Page
	Greek pi	24
	Russian peh	25
17	⠠⠠⠠⠠⠠ (dots 1-2-3-4-5)	
	English q	22
	German koo	23
	Greek koph (or qoph)	26
	Hebrew koph	24
	Russian cheh	25
18	⠠⠠⠠ (dots 1-2-3-5)	
	English r	22
	German err	23
	Greek rho	24
	Hebrew resh	25
	Russian err	25
	re (real part)	120
19	⠠⠠⠠⠠ (dots 2-3-4)	
	English s	22
	German ess	23
	Greek sigma	24
	Hebrew samekh	24

19	(Cont.)	Page
	Russian ess	25
	sec (secant)	120
	sech (hyperbolic secant)	120
	sin (sine)	120
	sinh (hyperbolic sine)	120
	sup (supremum)	120
20	(dots 2-3-4-5)	
	English t	22
	German teh	23
	Greek tau	24
	Hebrew teth	24
	Russian teh	25
	tan (tangent)	120
	tanh (hyperbolic tangent)	120
21	(dots 1-3-6)	
	English u	22
	German oo	23
	Greek upsilon	24
	Russian oo	25

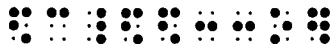
22		(dots 1-2-3-6)	English v	22
			German fao	23
			Greek vau	26
			Hebrew veth	24
			vers (versine)	120
23		(dots 1-3-4-6)	English x	22
			German iks	23
			Greek xi	24
			Hebrew cheth	24
			Russian shchah	25
24		(dots 1-3-4-5-6)	English y	22
			curved right full arrowhead	146
			German ypsilon	23
			Greek psi	24
25		(dots 1-3-5-6)	English z	22
			German tset	23
			Greek zeta	23

25		(Cont.)	Page
		Hebrew zayin	23
		Russian zeh	25
26		(dots 1-2-3-4-6)	
			
		curved left full arrowhead	{ 146
		factorial	! 152
		Greek chi	24
27		(dots 1-2-3-4-5-6)	
			
		blunted left full arrowhead	[146
		blunted right full arrowhead	] 146
		general omission symbol	70
28		(dots 1-2-3-5-6)	
			
		left parenthesis	(122
29		(dots 2-3-4-6)	
			
		Greek stigma	23
		Hebrew tsadi	152
		single integral	$\int$ 25
		Russian yerih	
		double integral	$\iint$ 152
		triple integral	$\iiint$ 153

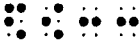
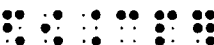
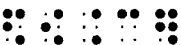
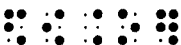
29	(Cont.)			Page
		integral with superposed circle	$\oint$	153
		integral with superposed rectangle	$\int_{\square}$	153
		integral with superposed square	$\int_{\square}$	153
		integral with superposed infinity	$\int_{\infty}$	153
30	(dots 2-3-4-5-6)			
		right parenthesis	)	122
31	(dots 1-6)			
		dot, and times	•	128, 129
		Hebrew chaph		23
		dot within inclusion sign	$\subseteq$	140
		dot within reverse inclusion sign	$\supseteq$	140
		dot between bars of equals sign	$\overline{=}$	140
32	(dots 1-2-6)			
		directly-over indicator (first order)		97
		index-of-radical indicator		108
		makes nearer arrowhead point up		145
		directly-over indicator (second order)		97
		upper limit	$\overline{\lim}$	119
		upper integral	$\overline{\int}$	153

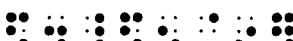
33	(dots 1-4-6)		Page
		directly-under indicator (first order)	97
		makes nearer arrowhead point down	145
		lower limit	<u>lim</u> 120
		lower integral	$\int$ 153
		directly-under indicator (second order)	97
34	(dots 1-4-5-6)		
		Greek theta	23
		Hebrew thav	24
		opening simple-fraction indicator	75
35	(dots 1-5-6)		
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		Hebrew sin	24
		horizontal bar (macron)	— 98
		Russian shah	25
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		bar over and bar under logical product	$\overline{\wedge}$ 138
		bar over and equals sign under logical product	$\overline{=}$ 138
		bar over single tilde	$\sim$ 139
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35	(Cont.)			Page
		bar over logical sum	$\overline{\vee}$	139
		bar over and bar under logical sum	$\overline{\underline{\vee}}$	139
		bar over and equals sign under logical sum	$\overline{=}$	139
		bar through inclusion sign	$\overline{\in}$	141
		bar through reverse inclusion sign	$\overline{\ni}$	141
		bar over inclusion sign	$\overline{\in}$	138
		bar over reverse inclusion sign	$\overline{\ni}$	138
		bar over less than sign	$\overline{<}$ or $\overline{<}$	138
		bar over greater than sign	$\overline{>}$ or $\overline{>}$	137
36	(dots 1-2-4-6)			
		Hebrew ayin		24
		Russian yah		25
		shape indicator		110
		arc concave upward	$\frown$	110
		circle	$\bigcirc$	111
		circle with interior dot	$\bigcirc \cdot$	112
		circle with interior arrow pointing up	$\bigcirc \uparrow$	112
		circle with interior arrow pointing up followed by interior arrow pointing down	$\bigcirc \uparrow \downarrow$	112
		circle with interior arrow pointing down	$\bigcirc \downarrow$	112
		circle with interior arrow pointing down followed by interior arrow pointing up	$\bigcirc \downarrow \uparrow$	112

36	(Cont.)			Page
		circle with interior arrow pointing left		112
		circle with interior arrow pointing left over interior arrow pointing right		112
		circle with interior arrow pointing right		112
		circle with interior arrow pointing right over interior arrow pointing left		112
		circle with interior plus sign		112
		circle with interior minus sign		112
		circle with interior cross		112
		diamond		111
		ellipse (oval)		111
		parallelogram		111
		rhombus		111
		irregular hexagon		111
		intersecting lines		111
		is parallel to		111
		arrow barbed at right (contracted form)		98, 135
		right-pointing arrow (contracted form)		110
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		isosceles triangle		113
		obtuse triangle		113
		right triangle		114
		scalene triangle		114
		trapezoid		111
		arrow dotted at left (no barb)		98
		arrow dotted at left and barbed at right		98
		arrow dotted at both ends		98
		vertical two-way arrow		135
		arrow pointing up		135
		arrow pointing up followed by arrow pointing down		140
		arrow pointing up followed by boldface arrow pointing down		140
		boldface arrow pointing up followed by arrow pointing down		140
		boldface arrow pointing up followed by boldface arrow pointing down		140
		arrow pointing down		135
		arrow pointing down followed by arrow pointing up		140
		arrow pointing down followed by boldface arrow pointing up		140
		boldface arrow pointing down followed by arrow pointing up		140

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		arrow barbed at left and dotted at right		98
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		adjacent angles		113
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36	(Cont.)			Page
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		angle with interior clockwise arrow		112
		angle with interior counterclockwise arrow		112
		short arrow pointing right over long arrow pointing left		137
		arrow barbed at right (uncontracted form)		98
		arrow pointing right over arrow pointing left		137
		arrow pointing right over boldface arrow pointing left		137
		arrow dotted at right (no barb)		98
		long arrow pointing right over short arrow pointing left		137
		arrow with upper barb pointing right over arrow with lower barb pointing left		137
		arrow with hollow dot at right (no barb)		98
		square		111
		square with interior dot		113
		square with interior horizontal bar		113
		square with interior vertical bar		113
		square with interior northwest- southeast diagonal		113
		square with interior diagonals		112
		square with interior southwest- northeast diagonal		113
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		regular hexagon		111

36	⠠⠠⠠ (Cont.)			Page
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	⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠	boldface arrow pointing left over arrow pointing right	↔	137
	⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠	boldface arrow pointing left over boldface arrow pointing right	↔	137
	⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠	boldface arrow pointing right over arrow pointing left	↔	137
	⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠	boldface arrow pointing right over boldface arrow pointing left	↔	137
	⠠⠠⠠ ⠠⠠⠠	filled-in square	■	130
	⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠	arrow with hollow dot at left (no barb)	⤵	98
	⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠	arrow with hollow dot at left and barbed at right	⤵	98
	⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠	arrow with hollow dot at both ends	⤵	98
37	⠠⠠⠠ (dots 1-2-4-5-6)			
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38	⠠⠠⠠ (dots 1-2-5-6)			
	⠠⠠⠠	Russian yu		25
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		straight right full arrowhead	⤵	146
		vertical bar as a sign of grouping		123
		vertical bar (is a factor, divides)		130
		vertical bar (such that)		136
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38	 (Cont.)		Page
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		vertical bar through shaft of arrow pointing right	 141
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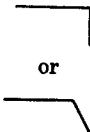
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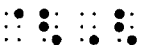
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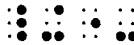
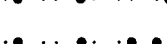
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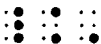
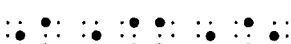
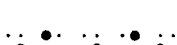
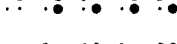
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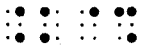
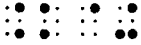
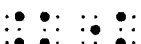
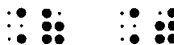
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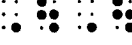
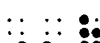
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